(0,2) Dynamics From Four Dimensions

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Main Idea

- Given a 4d N=1 supersymmetric theory with a global U(1) symmetry, we can associate a 2d model with (0,2) SUSY to it, as follows.

- Couple the current supermultiplet for the global U(1) to a non-dynamical external vector superfield.

- Then compactify the 4d theory on T^2, and turn on an expectation value for the magnetic field through the torus.

- This will actually break SUSY completely, as one can see from the variation of the gaugino in the external vector multiplet: $\delta\lambda = (F_{\mu\nu}\sigma^{\mu\nu} + iD)\epsilon$

- However, we can restore half the supersymmetry by turning on a background D-field with $D = B$.

$$\delta\lambda = i\begin{pmatrix} D-B & 0 \\ 0 & D+B \end{pmatrix}\begin{pmatrix} \epsilon_- \\ \epsilon_+ \end{pmatrix}$$

- The study of 2d (0,2) theories so obtained from 4d and their quantum properties — in particular, whether they preserve quantum (0,2) SUSY, and exhibit features of the 4d parent theories such as Seiberg duality — is the main topic of this talk.

Plan

- Free fields on a magnetized torus
- (Classical) N=1 SQCD on a magnetized torus
- Brane picture
- Quantum effects and (0,2) Seiberg duality
- Open puzzles

Free fields in a magnetic field

- Consider a free massless scalar ϕ with charge e under a U(1) gauge field A_μ. Turn on a background magnetic field, $A_2 = Bx_1$.

- The KG equation then takes the form

$$(-\partial_0^2 + \partial_3^2 + \partial_1^2 + \tilde{D}_2^2)\phi = 0, \qquad \tilde{D}_2 = \partial_2 + ieBx_1.$$

- Make the ansatz $\phi(x^0, x^1, x^2, x^3) = \varphi(x^0, x^3)\chi(x^1, x^2)$ and take χ to be an eigenfunction of

$$H = -(\partial_1^2 + \tilde{D}_2^2) = p_1^2 + (p_2 + eBx_1)^2; \qquad H\chi = m^2\chi.$$

- This is the Hamiltonian of a particle in a magnetic field. It has a discrete, massive spectrum $m_n^2 = (2n+1)|eB|$.

- Turning on a vev for the background D, as required in order to preserve SUSY, shifts the spectrum to

$$m_n^2 = (2n+1)|eB| - eD.$$

- For $B = D > 0$, there are massless fields when $e > 0$.

- Fields with $e < 0$ give rise to a massive spectrum.

- There's a similar story for free fermions charged under a magnetic field. Right-moving ones have a spectrum aligned with the scalars, while the left-moving ones don't:

$$m_+^2 = (2n+1)|eB| - eB, \qquad m_-^2 = (2n+1)|eB| + eB$$

- This is consistent with (0,2) supersymmetry.

- To summarize: a 4d free massless chiral superfield with charge e reduces under a constant $B = D > 0$ field to

  - $e > 0$: massless (0,2) chiral superfield.

  - $e < 0$: massless (0,2) Fermi superfield.

  - There are also an infinite tower of massive fields with masses of order B.

Compactification

- Next, we take the coordinates $x^{1,2}$ to be periodic.
- This leads to Dirac quantization $eB\mathcal{A} \in 2\pi Z$ for the background B-field, with $\mathcal{A}$ the area of the torus.
- The spectrum is the same, but states have degeneracy $n_e = |e|B\mathcal{A}/2\pi$.
- We can normalize them so a field of charge e has degeneracy $|e|$.

4d N=1 SQFT on Magnetized Torus

- We now generalize to the interacting case, starting from a 4d N=1 SUSY gauge theory with charged matter.
- To apply our procedure, we pick a non-anomalous global U(1) symmetry of the theory, and turn on a background B and D field for it.
- We take the U(1) to be orthogonal to the gauge group.

- The resulting 2d massless field content is the following.
  - A (0,2) gauge superfield Υ.
  - An adjoint chiral superfield Σ, from reducing the gauge multiplet.
  - e_i (0,2) chiral superfields for each matter field with $e_i > 0$
  - $|e_i|$Fermi superfields for each with $e_i < 0$
  - Both a chiral and Fermi superfield for each with $e_i = 0$.

- The exact 2d (0,2) Lagrangian can in principle be computed by integrating out massive modes, where the matter fields have non-standard wavefunctions in the compact directions because of the background fields.

- Integrating out massive modes introduces interactions between the massless ones. In particular, the Lagrangian will generically contain Kahler terms of the schematic form

$$\int d^2\theta|\Sigma|^2\bar{\Phi}D_-\Phi, \int d^2\theta|\Sigma|^2|\Lambda|^2, \int d^2\theta|\Phi^2|\Lambda|^2...$$

Example: N=1 SQCD

- Gauge group $U(N_c)$
- N_f flavors $Q, \tilde{Q}$ in the (anti)fundamental of the gauge group.
- Global non-R symmetry is $SU(N_f) \times SU(N_f)$
- To apply our procedure, we pick an anomaly-free subgroup of the global symmetry.

- We'll focus on the example of the background U(1) that assigns charges +1 to half the $Q's$ and -1 to the other half, similarly for the $\tilde{Q}'s$.

- Following the rules laid out earlier, the resulting (0,2) theory contains $N_f/2$ chiral superfields and $N_f/2$ Fermi superfields in the (anti)fundamental.

Seiberg Duality

- 4d SQCD has the property that its IR limit is equivalent to that of a different theory with gauge group $U(N_f - N_c)$, N_f chiral multiplets $q, \tilde{q}$ in the (anti)fundamental, and a gauge singlet chiral superfield M^i_j coupled to the quarks through a superpotential $\mathcal{W} = Mq\tilde{q}$.

- A natural question is if Seiberg duality holds for the (0,2) theories obtained by our procedure.

- The 4d gauge theories we start from are asymptotically free and develop strong coupling in the IR below a dynamically generated scale Λ.

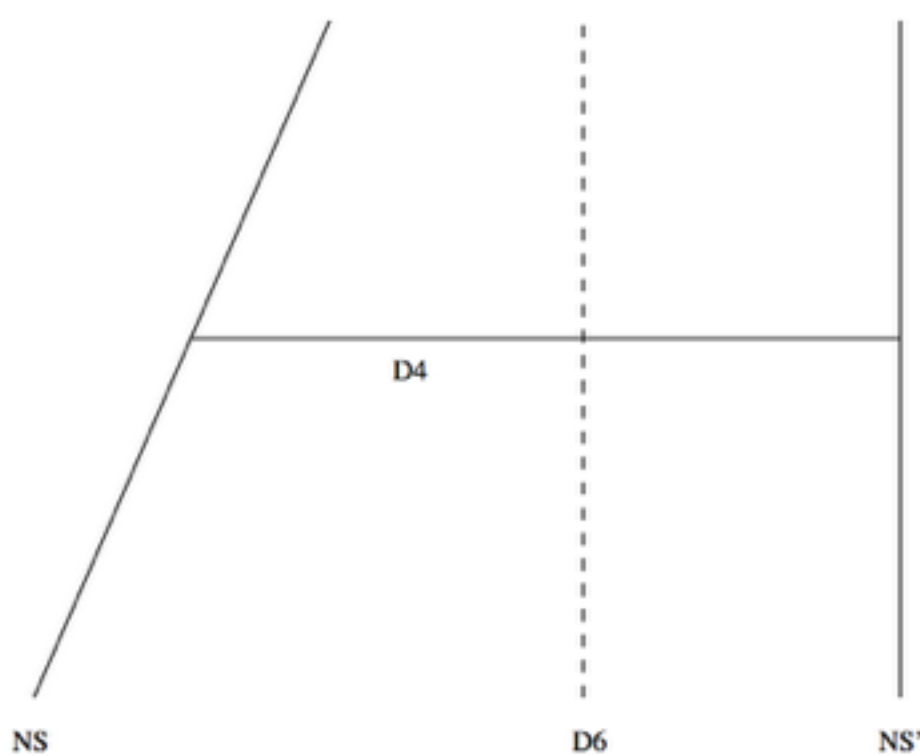
- Our procedure introduces new scales from the strength of the B-field and the size of the two-torus.

- If these scales are much larger than Λ, they modify the dynamics when the 4d theory is still weakly coupled, and our procedure is expected to go through.

- If the hierarchy of scales is the other way around, we naively cannot use the procedure. However, as the two regimes are connected by a continuous deformation (the size of the torus), and we do not expect there to be a phase transition, we nevertheless expect to find a 2d duality.

Brane Construction

- 4d N=1 SQCD can be embedded on a system of intersecting D-brane and NS5-branes.



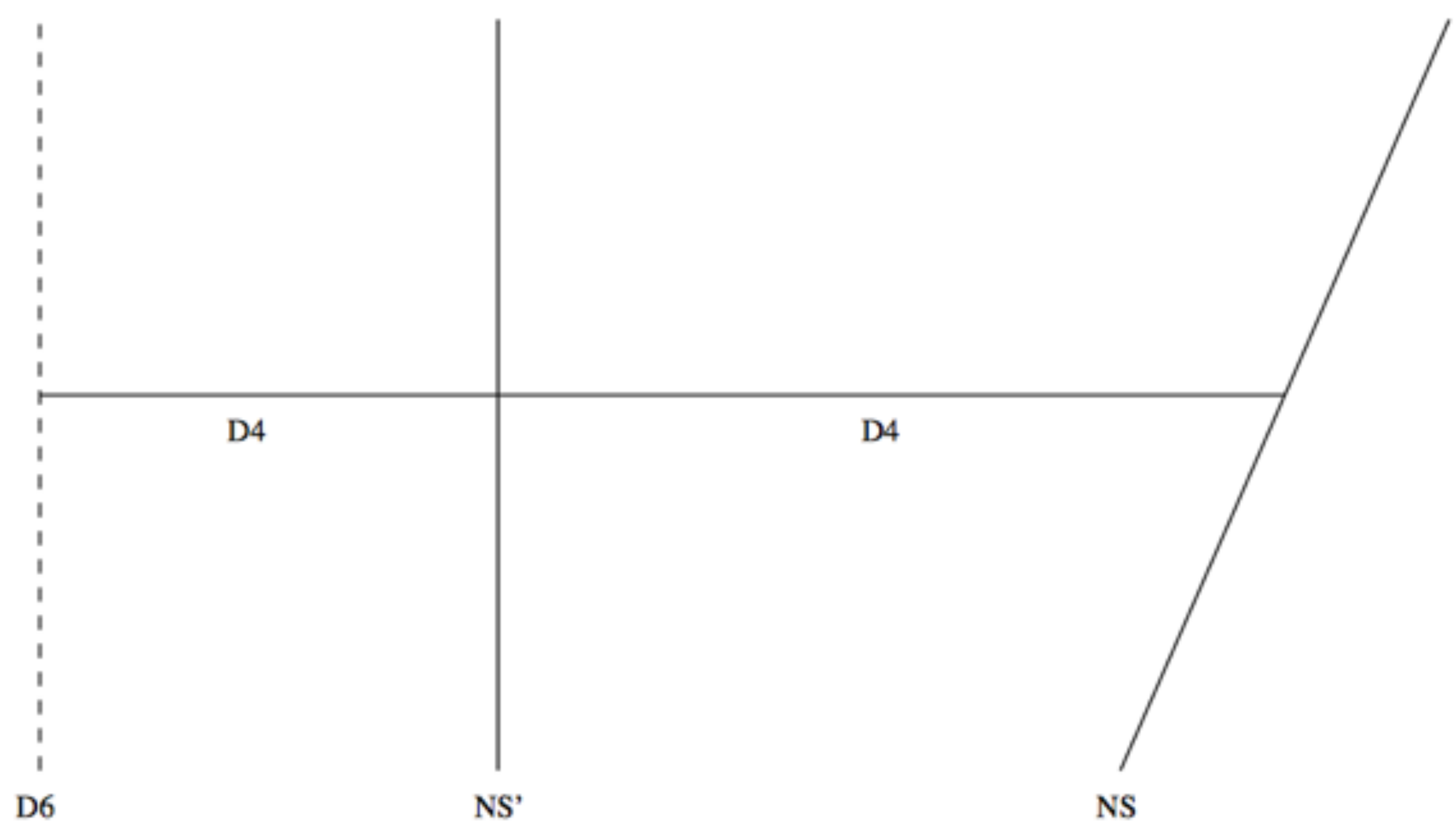
$$N_c D4's : 01236$$

$$N_f D6's : 0123789$$

$$NS : 012345$$

$$NS' : 012389$$

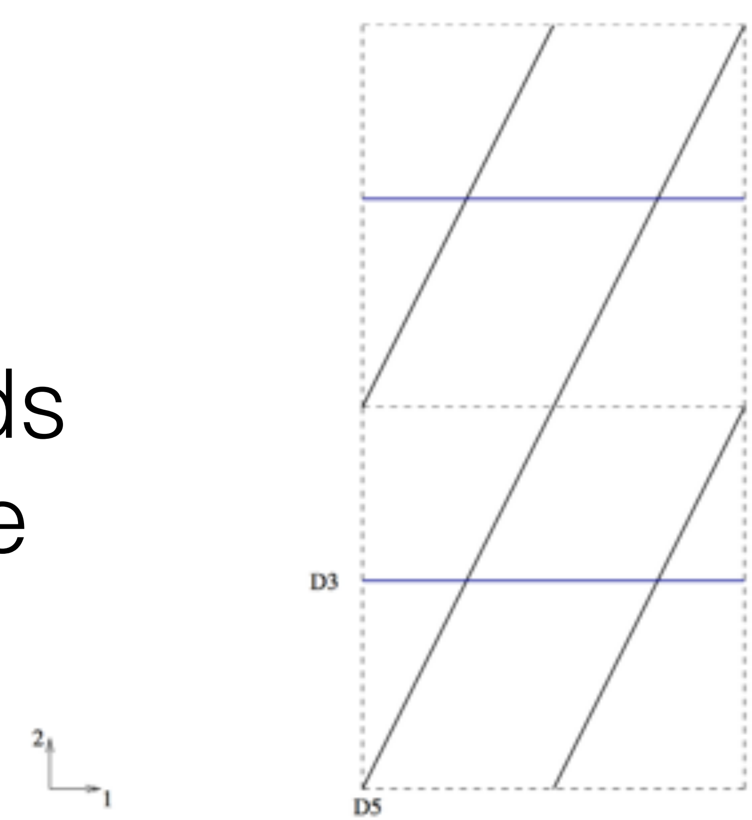
- The Seiberg dual in the brane picture is given by the following setup.



- In the 4d brane picture, the B-field for the background U(1) in our procedure is implemented by turning on B-fields for the U(1) on the worldvolume of the D6 branes. The D-term comes from rotating the D6 branes in the (67) plane.

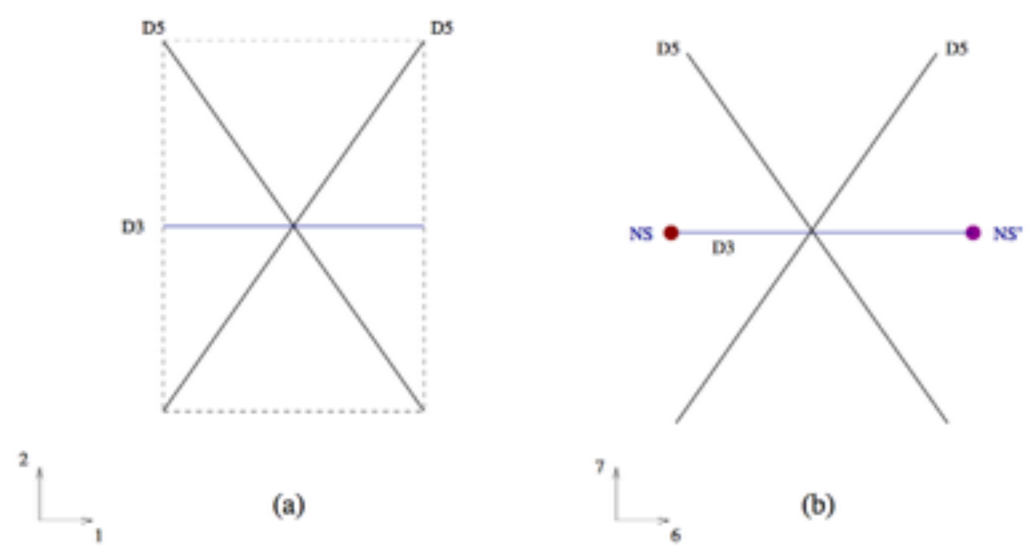
- It would be nice to geometrize the B-field as well. To this end we T-dualize in say the x^2 direction, and move to a 3d brane picture.

- A B-field on a D6 brane worldvolume, now corresponds to rotating the T-dual D5 brane in the 12 plane.



- In the 3d picture, lots of things are manifest.
  - Quantization of the B-field
  - The degeneracy of $|e|$
  - It is now manifest that 1/2 SUSY is preserved in our construction: this is the well-known fact that 1/2 SUSY is preserved under brane rotation in two planes (17)->(26) by equal angles.

- When we start with 4d N=1 SQCD and choose for our procedure the global U(1) that assigns charge 1 to $N_f/2$ of the $Q's$, -1 to the other half, and likewise for the $\tilde{Q}'s$, the brane picture looks like the following.



(a) (b)

The Quantum Theory

- The light states of the 2d theory from our procedure are the following.

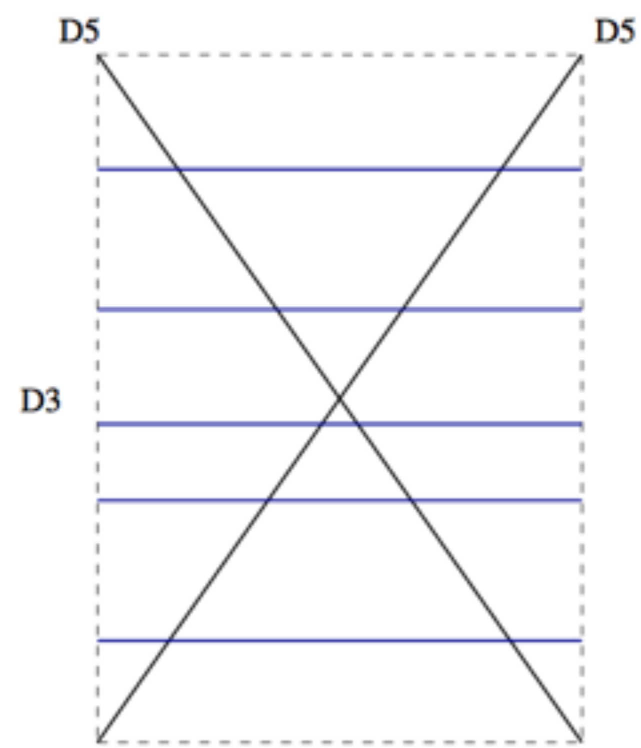
field	$SU(N_f/2)_1$	$SU(N_f/2)_2$	$SU(N_f/2)_3$	$SU(N_f/2)_4$	$U(1)_e$
Q^1	$N_f/2$	1	1	1	$+1$
Λ^2	1	$N_f/2$	1	1	-1
$\widetilde{\Lambda}_1$	1	1	$\overline{N_f/2}$	1	-1
$\widetilde{Q}_2$	1	1	1	$\overline{N_f/2}$	$+1$

- There are also the (0,2) adjoint fields Υ, Σ from the reduction of the 4d vector multiplet.

Coulomb branch

- The theory has a classical Coulomb branch parametrized by eigenvalues of σ. Geometrically, it corresponds to displacing the D3's in the x^2 direction.

- Classically, the Coulomb branch potential is flat.



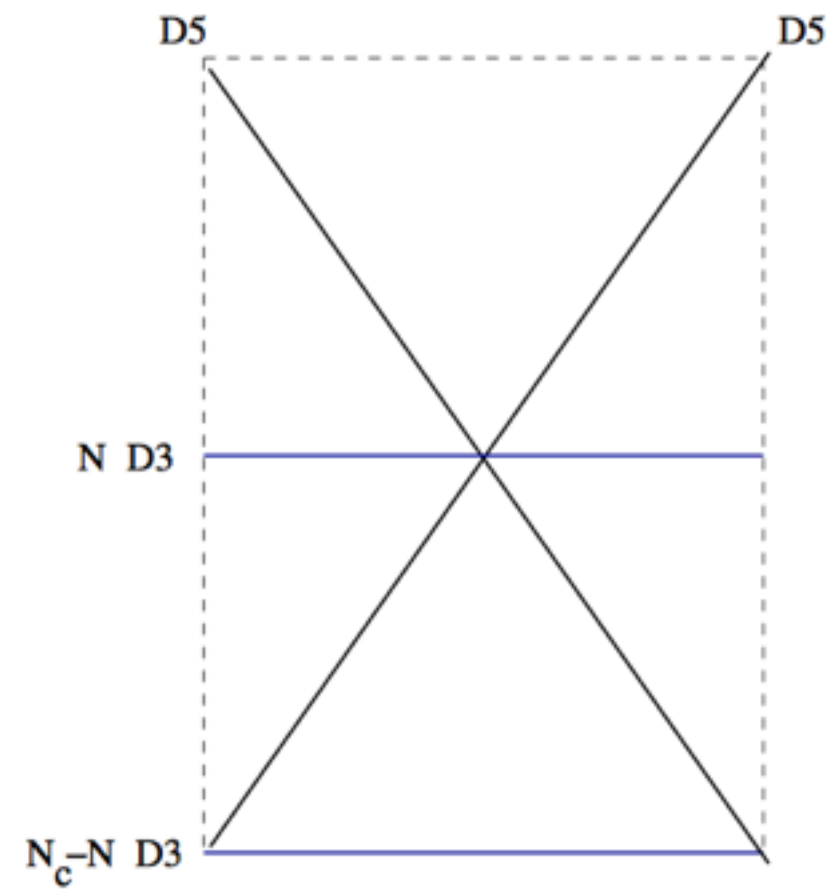
- However, the Coulomb branch is lifted by quantum effects and replaced by a discrete set of vacua.

- At a generic point on the Coulomb branch, the (0,2) chiral superfields $Q^1, \tilde{Q}_2$ with charge ± 1 under the gauge group, are localized at different points on the x^1 circle.

- Locally in x^1, the theory looks like two copies of a U(1) gauge theory, one with just Q^1 and the other with just $\tilde{Q}_2$ as the (0,2) chiral superfields.

- Such theories are known to dynamically break (0,2) SUSY.

- The Coulomb branch is replaced by discrete vacua labeled by an integer N, which in the brane picture corresponds to the number of D3 branes at one of the two intersections.



- At the upper intersection, the bosonic field content looks like that of SQCD with gauge group $U(N)$ and $N_f/2$ flavors. (Similarly for the lower intersection).

- We assume that the stability bound for each vacuum is the same as in 4d SQCD. Taking into account the bound from both intersections gives

$$\max(0, N_c - \frac{1}{2}N_f) \leq N \leq \min(N_c, \frac{1}{2}N_f) \ .$$

- In each of the two copies, the moduli space is a σ model on the space of solutions to the D-term equations. Going to large $Q, \tilde{Q}$, we can read off

$$c_R = c_L = 3(N_f N - N^2) \ .$$

(0,2) Seiberg duality

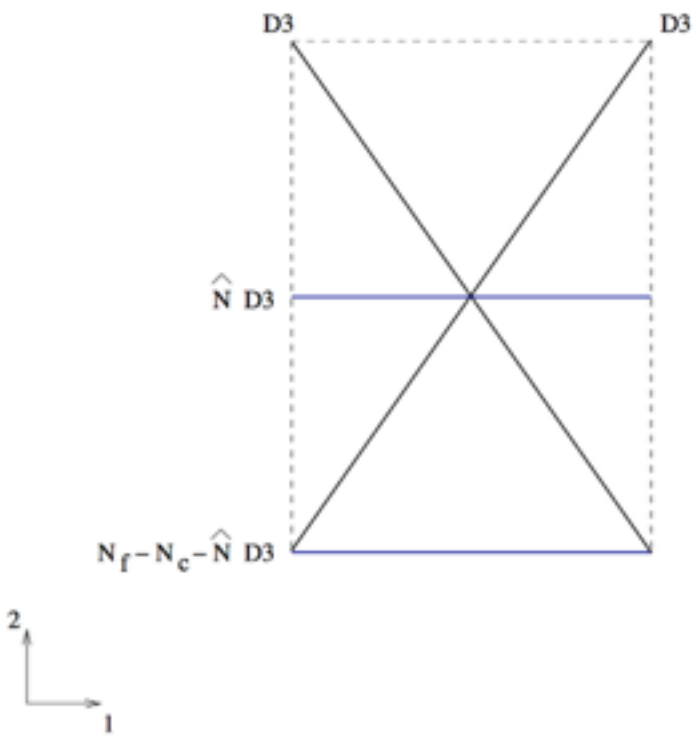
- The Seiberg dual theory with gauge group $U(N_f - N_c)$, contains the following light fields after the reduction with the background field.

field	$SU(N_f/2)_1$	$SU(N_f/2)_2$	$SU(N_f/2)_3$	$SU(N_f/2)_4$	$U(1)_e$
λ_1	$\overline{N_f/2}$	1	1	1	-1
q_2	1	$\overline{N_f/2}$	1	1	$+1$
$\widetilde{q}^1$	1	1	$N_f/2$	1	$+1$
$\widetilde{\lambda}^2$	1	1	1	$N_f/2$	-1
M_1^1, Λ_1^1	$N_f/2$	1	$\overline{N_f/2}$	1	0
M_2^2, Λ_2^2	1	$N_f/2$	1	$\overline{N_f/2}$	0
$M_2^1(\times 2)$	$N_f/2$	1	1	$\overline{N_f/2}$	$+2$
$\Lambda_1^2(\times 2)$	1	$N_f/2$	$\overline{N_f/2}$	1	-2

- There's also an effective (0,2) superpotential

$$\mathcal{W} = M_1^1 \lambda_1 \widetilde{q}^1 + M_2^2 q_2 \widetilde{\lambda}^2 + \Lambda_1^2 q_2 \widetilde{q}^1 \,.$$

- In this case, there is a similar brane picture.



- The color branes are again localized at the two intersections, with discrete vacua labeled by

$$\max(0, \frac{1}{2}N_f - N_c) \leq \widehat{N} \leq \min(N_f - N_c, \frac{1}{2}N_f) \ .$$

- There are the same number of vacua in the two sides.

- By comparing 't Hooft anomalies for the $SU(N_f/2)^4$ global symmetries, we find the map between the parameters labeling electric and magnetic vacua,

$$\hat{N} = \frac{N_f}{2} - N.$$

- Generalizations: instead of taking half the Q's to have charge 1 and the other have charge -1, assign to them arbitrary (anomaly-free) charges e_i.

- Generic $e_i's$ break SUSY dynamically. Consider e.g. $e_1 = 2, e_2 = e_3 = -1$. It's easy to see from the brane picture that at intersections there is an incomplete cancellation of charge between $Q, \tilde{Q}$. This is the typical scenario.

Comparison of R-charge

- As a check on our picture, we can attempt to match the $U(1)_R$ anomaly of the nominal dual theories.

- On the electric side,

$$\begin{aligned} Q, \tilde{Q} &: 0 \\ \Lambda, \tilde{\Lambda} &: 0 \\ \Sigma &: 1 \\ \Upsilon &: 1 \end{aligned}$$

- This gives $k = 2 \times \frac{1}{2} N_f N - N^2$ which is a third of the central charge, as expected.

- On the magnetic side, we have

$$\Sigma : 1$$

$$\Upsilon : 1$$

$$M^1_2 = Q^1 \tilde{Q}_2 : 0$$

- The magnetic superpotential determines the R-charge of the other fields in terms of $R(q)$.

- If we set the $U(1)^2_R$ anomaly of the electric and magnetic theories equal to find $R_q = (N - \hat{N})/N_f$, this gives agreement of the (independent and nonzero) $U(1)_R U(1)_e$ anomaly.

- However, we do not understand how to obtain the R-charge assignments directly in the magnetic theory.

(0,2) index

- Another puzzle is the matching of the (0,2) indices in the two sides of the proposed duality.

- The 2d (0,2) index is a refined Witten index in radial quantization, defined by

$$\mathcal{I} = \mathrm{Tr}(-1)^F q^{H_L} a^f$$

- where H_L is the left-moving dilation generator, a parametrizes fugacities for the flavor symmetries, and the trace is over states with $\{Q_+, Q_+^\dagger\} = 0$.

- It's known how various chiral matter fields contribute to the index:

$$I_\Phi = \theta(q^{R/2}a;q)^{-1}, \qquad I_\Psi = \theta(q^{(R+1)/2}a;q), \qquad \theta(a;q) = \prod_{i=0}^{\infty}(1-aq^i)(1-q^{i+1}/a)$$

- The gauge field contributes additional θ functions that depend on the fugacities for the Cartan of the gauge group.

- To compute the index of a (0,2) gauge theory that flows to a SCFT, we multiply the contribution of all multiplets and integrate over the gauge group fugacities to impose the Gauss law.

- When we naively compute the indices of the electric and magnetic theories in our proposed dualities, they do not match.

- The source of the problem is that the magnetic theory contains a (0,2) Fermi superfield Λ_1^2 transforming in the (anti)fundamental of $SU(N_f/2)_2(SU(N_f/2)_3)$.
- It's not obvious how to map this operator to one in the chiral ring of the electric theory.

- As mentioned earlier, our Lagrangian contains $\int d^2\theta |\Sigma|^2 |\Lambda|^2$ terms.

- The component expansion gives current-current couplings $J_+ J_-$ between

$$J_+ = \bar{\sigma} D_+ \sigma + \bar{\lambda}_+ \lambda_+$$
$$J_- = \bar{\psi}_- \psi_- .$$

- The index doesn't see such terms; however there are 2d examples where similar current-current interactions change the dynamics and break would-be symmetries.

Summary

- We conjectured that 4d Seiberg dualities descend to dualities of non-Abelian gauge theories in two dimensions with (0,2) SUSY.
- However, there are still some open issues.