Supergravity duals of the simplest Chern-Simons matter theories

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Appearing tonight
Duals for a 3d superconformal theory with a single gauge group.

Dyonic ISO(7) gauged supergravity and its massive IIA uplift.

Exact $N=2$ AdS$_4$ massive IIA solutions.

A beautiful agreement!
In gauged maximal supergravity in four dimensions, one can adjust the combination of electric and magnetic vector fields that are gauged. [G. Dall’Agata, G. Inverso and M. Trigiante]

In the interacting nonabelian theory, these result in different theories – it cannot be undone by an overall electromagnetic duality.
Are these theories quantum consistent?

- The electric SO(8) gauging of 4d maximal supergravity is the 4d massless truncation of M-theory on S^7.

- There is no known candidate for a string theory completion of the dyonic theories.  
  [Bernard de Wit, Hermann Nicolai]

- In AdS_4, boundary conditions break SUSY to N=3, so no hint from N=8 3d SCFTs.  
  [A. Borghese, Y. Pang, C.N. Pope, E. Sezgin]
ISO(7)

- Contraction of SO(8) to $SO(7) \times R^7$. [Hull, Hull Warner]

- Only the 7 translations can be magnetically gauged. This will be a consistent truncation of massive IIA on $S^6$. The Romans mass determines the magnetic coupling.

- Generalizes the magnetic gauging induced by the Romans mass in Calabi-Yau compactifications. [Polchinski Strominger]
New AdS$_4 \times S^6$ solutions

- The exact metric and fluxes can be found using a systematic uplifting of an AdS critical point of the 4d ISO(7) dyonically gauged supergravity.

- The sphere is squashed in these cohomogeneity 1 solutions in 10d.

- There are solutions with $N=1,2,3$ SUSY.
D2 brane physics

- In flat space, the decoupled theory on N D2 branes is the maximal 3d Yang-Mills theory.

- Flows to the M2 brane SCFT. The 7 scalars are completed by 1 that appears due to monopole condensation, filling out the SO(8) multiplet.

- Strongly coupled, and can’t even compute susy quantities from the UV YM Lagrangian.
A 2004 idea for CSM/AdS$_4$

- The most obvious way to get a CFT is to add Chern-Simons terms. Then the 3d gauge theory looks classically conformal.

- The problem, noted by Schwarz is that one gets at most $N=3$ SUSY. Moreover, the CS term seems to be induced by the Romans mass.

- Until now, no adjoint SCFT of this type had a known gravity dual.
Review of ABJM

- The idea was to relate M-theory to IIA along a different “Hopf” circle that shrinks at the origin.

- This eliminated the smearing of the M2 branes in the D2 uplift. Hence we obtained a Lagrangian that didn’t flow to infinite coupling and undergo monopole condensation.

- The price was that the IIA geometry is a non-trivial cone over CP³. Led to a product gauge group.
Adding the Romans mass

- Adding the $F_0$ flux to that setup in the ‘t Hooft limit gives a theory with a total CS level,
  \[ m = \hat{F}_0 = k/(2\pi \ell_s) \]  
  in IIA on $\text{AdS}_4 \times \mathbb{CP}^3$.

- The $N=2$ solution with $\text{SO}(4)$ isometry was found up to solving non-linear ODEs.

- Curiously remains weakly coupled in the “M-theory” limit.
Theories with one gauge group?

- Most CS theories with adjoint matter cannot have gravity (rather than stringy) duals.

- For $N \geq 2$, there are light protected higher spin operators and a Hagedorn growth in the spectrum for 3 or more adjoint chirals.

- With only a single chiral, large ‘t Hooft coupling is not allowed in the CFT.
ISO(7) dyonically gauged $N=8$ supergravity

- Defined using the embedding tensor formalism.

- The SO(7) is electrically gauged, while the translations are dyonic. At the supergravity level, there are only two discrete choices.

- 70 scalars $\mathcal{M}_{IJ}$ of the $E_{7(7)}/SU(8)$ coset, and field strengths for the $R^7$ modified to

\[ F^I = dA^I - g\delta_{JK} A^{IJ} \wedge A^K + mA^{IJ} \wedge \tilde{A}_J - m \delta^{IJ} B_J \]
A simple truncation

- Keeping only SU(3) singlets of the ISO(7) group and the SU(8) R symmetry group one obtains an N=2 subsector.

- One can further truncate to just the metric and three scalars.

\[ \mathcal{L} = R - 2(\partial \phi)^2 - \frac{3}{2} (\partial \varphi)^2 - \frac{3}{2} e^{2\varphi} (\partial \chi)^2 - V \]

\[ V = \frac{1}{2} g^2 \left( e^{4\phi - 3\varphi} (1 + e^{2\varphi} \chi^2)^3 - 12 e^{2\phi - \varphi} (1 + e^{2\varphi} \chi^2) - 24 e^{\varphi} \right) - g m e^{4\phi + 3\varphi} \chi^3 + \frac{1}{2} m^2 e^{4\phi + 3\varphi} \]

- The potential depends on m.
**New 4d supergravity solutions**

- This sector has three AdS critical points. Two are nonsupersymmetric and unstable.

- The third is new, has $N=2$ supersymmetry, and $SU(3) \times U(1)$ isometry.

\[
e^{6\phi} = \frac{64}{27} g^2 m^{-2}, \quad e^{6\phi} = 8 g^2 m^{-2}, \quad \chi^3 = -\frac{1}{8} g^{-1} m
\]

- We will find the exact 10 uplift and its CFT dual.
An uplift formula

- The 10d theory is written with only SO(3,1) manifest, such that the supersymmetry variations can conform to an E\(_{7(7)}\) vector-tensor hierarchy.
- Exact nonlinear uplift ansatz verified by checking that all S\(^6\) data drops out of the susy variations. In work to appear by Oscar Varela and Adolfo Guarino.

\[ ds_{10}^2 = \Delta^{-1} ds_4^2 + \frac{1}{4}g^2 \Delta K^m_{IJ} K^n_{KL} M^{IJ KL} D y_m D y_n \]

\[ D y^m = d y^m + \frac{1}{2} g K^m_{IJ} A^{IJ} \]

where K are the Killing vectors of the round 6-sphere
The exact 10d solution

- Hopf squashed 5 sphere fibered over an interval:

\[ d\hat{s}_{10}^2 = L^2 (3 + \cos 2\alpha)^{1/2} (5 + \cos 2\alpha)^{1/8} \left[ ds^2(\text{AdS}_4) + \frac{3}{2} d\alpha^2 + \frac{6 \sin^2 \alpha}{3 + \cos 2\alpha} ds^2(\mathbb{C}P^2) + \frac{9 \sin^2 \alpha}{5 + \cos 2\alpha} \eta^2 \right], \]

\[ e^\hat{\phi} = e^{\phi_0} \frac{(5 + \cos 2\alpha)^{3/4}}{3 + \cos 2\alpha}, \]

\[ L^{-2} e^{-\frac{1}{2} \phi_0} \hat{H}_{(3)} = 24\sqrt{2} \frac{\sin^3 \alpha}{(3 + \cos 2\alpha)^2} J \wedge d\alpha, \]

\[ L^{-3} e^\frac{1}{4} \phi_0 \hat{F}_{(4)} = 6 \text{vol}(\text{AdS}_4) + 12\sqrt{3} \frac{7 + 3 \cos 2\alpha}{(3 + \cos 2\alpha)^2} \sin^4 \alpha \text{vol}(\mathbb{C}P^2) + 18\sqrt{3} \frac{(9 + \cos 2\alpha) \sin^3 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} J \wedge d\alpha \wedge \eta, \]

\[ L^{-1} e^\frac{3}{4} \phi_0 \hat{F}_{(2)} = -4\sqrt{6} \frac{\sin^2 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} J - 3\sqrt{6} \frac{(3 - \cos 2\alpha)}{(5 + \cos 2\alpha)^2} \sin \alpha d\alpha \wedge \eta \]

\[ m = \hat{F}_0 = k/(2\pi \ell_s), \quad L^2 = 2^{-\frac{5}{8}} 3^{-1} g^{-\frac{25}{12}} m^{\frac{1}{12}} \quad \text{and} \quad e^{\phi_0} = 2^{\frac{1}{4}} g^{\frac{5}{6}} m^{-\frac{5}{6}}. \]
Flux quantization

In the quantum theory, the Romans mass is quantized, as is the flux wrapping the compact 6-sphere.

\[ N = \frac{-1}{(2\pi \ell_s)^5} \int_{S^6} e^{\frac{1}{2} \phi} \hat{F}_4 + \hat{B}_2 \wedge d\hat{A}_3 + \frac{1}{6} \hat{F}_0 (\hat{B}_2)^3 \]

This fixes the supergravity parameters $g$ and $m$ in terms of the charges, which we will need for an exact comparison to the SCFT dual.
Evaluating the free energy

- The gravitational free energy, i.e., the value of on-shell action is given simply in four dimensions by $\pi/2G_N$.

- This can be calculated from the 10d action,

$$F = \frac{16\pi^3}{(2\pi \ell_s)^8} \int_{S^6} e^{8A} \text{vol}_6 = \frac{\pi}{5} 2^{1/3} 3^{1/6} N^{5/3} k^{1/3}$$

where $e^{2A}$ is the warp factor.
The dual field theory

- These backgrounds with $S^6$ internal manifold should arise as the near horizon geometry of N D2 branes in a smooth space. RR fluxes are required to preserve supersymmetry with $F_0 \neq 0$.

- Simply add $N=2$ (or 1,3) Chern-Simons terms to the maximal YM theory.

- Three adjoint chirals with $\mathcal{W} = \text{Tr}(X[Y, Z])$ SU(3) flavor symmetry and U(1) R symmetry.
Some rg flows

- Adding an F-term mass for one of the chirals results in a quartic superpotential, flows to the $N=3$ point with SU(2) flavor and SU(2) $R$ in the IR. Can find this in the gravity dual.

- From the $N=2$ Gaiotto Tomasiello theory, one can flow to ours on the moduli space – the off-diagonal gauge group is Higgsed.
S$^3$ free energy

- The gauge theory dual of the euclidean AdS free energy is $F = - \log Z$ on the euclidean three sphere.

- There is a well defined finite term for odd dimensional CFTs. These are mapped conformally to the sphere.

- One can preserve supersymmetry all along an R-symmetric rg flow (say via a UV Lagrangian).
Localization

- Supersymmetric path integrals are invariant under deformation by $Q$ variations.

- Can (sometimes) use this to reduce to an integral over supersymmetric configurations.

- Also implies agreement between UV and IR calculations, thus one can use the Lagrangian.
Exact partition function

The result is an integral over the eigenvalues of the adjoint scalar in the vector multiplet, with 1-loop determinants from the vector and chirals.

\[
Z = \int \prod_{i=1}^{N} \frac{d\lambda_i}{2\pi} \prod_{i<j=1}^{N} \left(2\sinh^2\left(\frac{\lambda_i - \lambda_j}{2}\right)\right) \times \\
\prod_{i,j=1}^{N} \left(\exp\left(\ell(1 - \frac{2}{3} + \frac{i}{2\pi}(\lambda_i - \lambda_j))\right)\right)^3 e^{\frac{ik}{4\pi} \sum \lambda_i^2} \\
\ell(z) = -z \log(1 - e^{2\pi i z}) + \frac{i}{2} \left(\pi z^2 + \frac{1}{\pi} \text{Li}_2(e^{2\pi i z})\right) - \frac{i\pi}{12}
\]
The large N limit should agree with supergravity.

The eigenvalues will form a density.

In the 't Hooft limit, one always finds $F = N^2 f(\lambda)$. Hard to solve here.

In the large N, fixed k limit, need some cancelations to have any saddle.
Range of validity of supergravity

- The radius of curvature of the string frame metric in string units scales like \((N/k)^{1/6}\), while the string coupling scales as \(\frac{1}{N^{1/6}k^{5/6}}\). Thus supergravity is valid when \(N\) is much greater than \(k\).

- The eigenvalue distribution grows with \(N\) in this limit, and there is a saddle only because the long range inter-eigenvalue interactions cancel.
The free energy

- Making an ansatz $\lambda = N^\alpha (x + iy(x))$, and writing the integral in terms of the density of eigenvalues, one finds a local effective action.

$$S = \frac{N^{1+2\alpha}}{4\pi} k \int dx \rho(x) \left( 2xy(x) - i(x^2 - y^2) \right) + \frac{32}{27} \pi^2 N^{2-\alpha} \int dx \frac{\rho^2(x)}{1+y'(x)}$$

- The saddle point equations are algebraic, and one easily finds that

$$F = \frac{3^{13/6}}{40} \pi \left( \frac{32}{27} \right)^{2/3} k^{1/3} N^{5/3}$$

in perfect agreement with supergravity!
Summary

- The $N=8$ dyonically gauged ISO(7) 4d supergravity is a consistent truncation of massive IIA on $S^6$, even though no $N=8$ solutions exist. Have explicit uplift formula.

- First exact $N=2$ AdS$_4$ solution in massive IIA.

- First example of a 3d CSM theory with a simple (single) gauge group dual to supergravity. Exact match of $F!$