Superstring Field Theory on the Small Hilbert Space

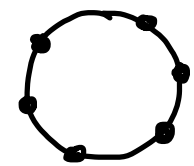
T. Erler, S. Konopka, I. Sachs '13, '14, to appear

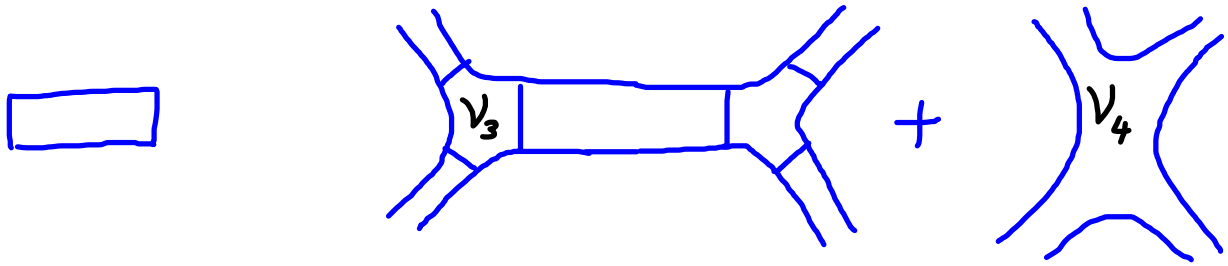
Plan:

- Review of bosonic OSFT
- Deformation theory
- NS-sector of superstring field theory
- R-sector

tree-level open (bosonic) SFT:

~ decomposition of the moduli space of disks with punctures





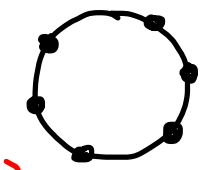
B-V structure: $\partial V_4 + (V_3 , V_3) = 0$

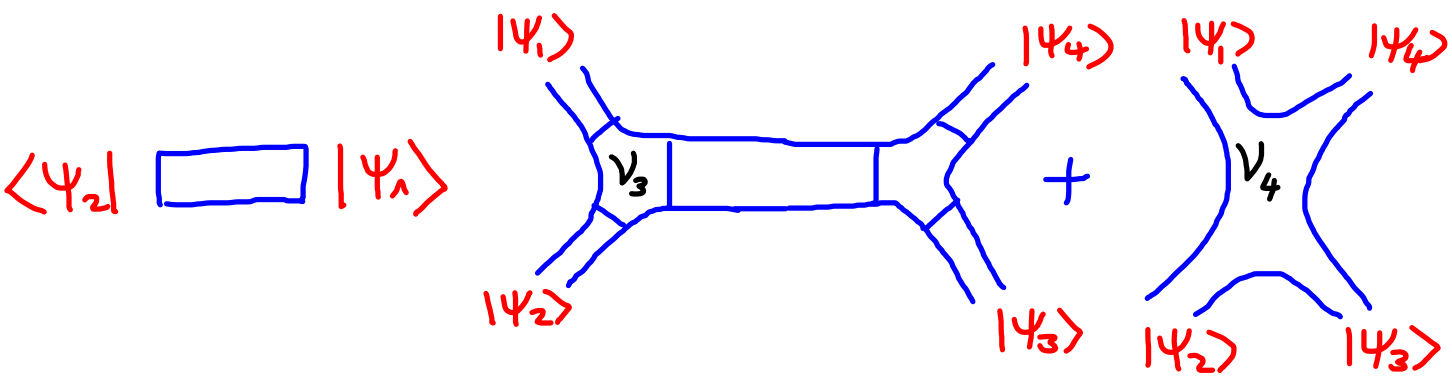
boundary operator

↳ glueing ~ 

tree-level open (bosonic) SFT:

~ decomposition of the moduli space of disks with punctures





$$\text{B-V:}\quad \partial V_4 + (V_3, V_3) = 0$$

$$\Rightarrow \quad (S,S) = 0$$

$$\overset{\shortparallel}{\omega^{a\bar{b}}\,\partial_a S\,\partial_{\bar{b}} S}$$

CFT defines a morphism between B-V algebras

CFT: $\mathcal{H}_0$, $\omega(\Psi_1, \Psi_2)$, $\omega(\Psi_1, \Psi_2 * \Psi_3)$

matter + ghost

↑

BRST

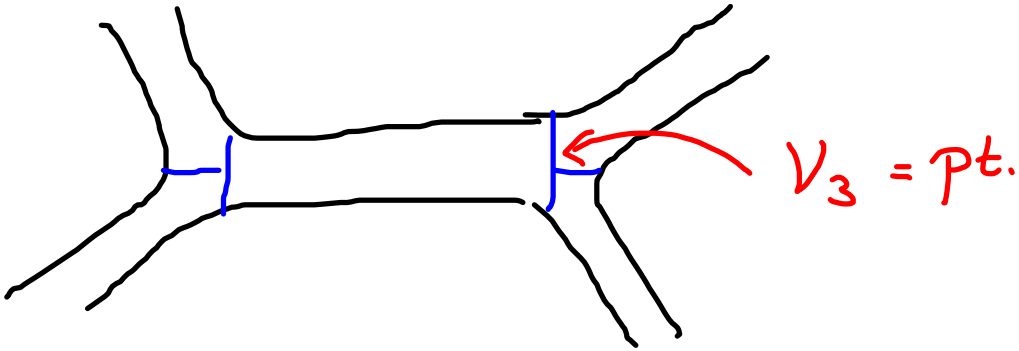
$\mathcal{H}_0 = \mathcal{H}_{CFT}$

$\omega(\Psi_1, \Psi_2) =$ BPZ inner product

$\omega(\Psi_1, \Psi_2 * \Psi_3) =$ 

Witten's OSFT: $S[\Psi] = \omega(\Psi, \underbrace{Q\Psi}_{m_1(\Psi)}) + \frac{g}{3!}\,\omega(\Psi, \underbrace{\Psi * \Psi}_{(-1)^{|\Psi|} m_2(\Psi,\Psi)})$

$|\Psi| = gh(\Psi) - 1$



$$\hat{m} = Q \otimes \mathbb{1} \otimes \mathbb{1} \cdots + m_2 \otimes \mathbb{1} \otimes \mathbb{1} \cdots$$

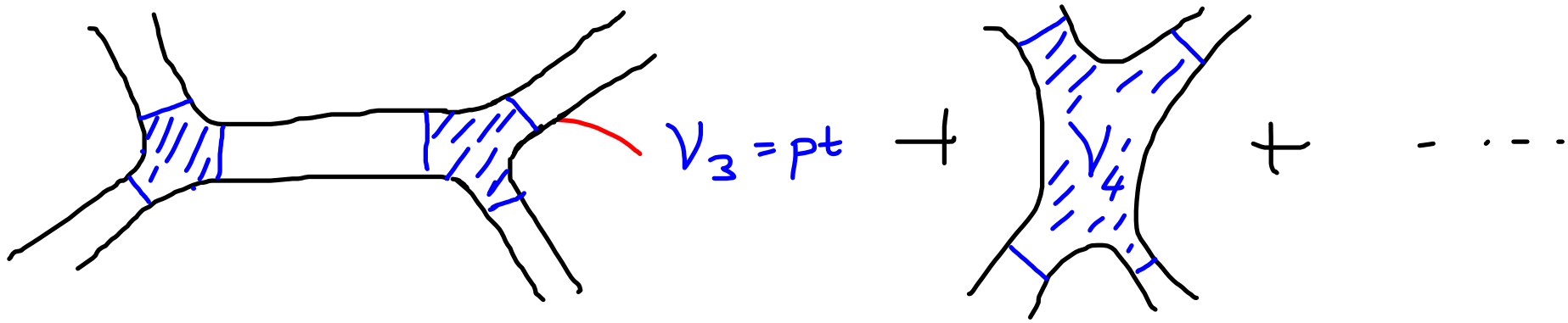
$$\hat{m} : \underbrace{\mathcal{H}_0 \otimes \mathcal{H}_0 \otimes \mathcal{H}_0 \otimes \mathcal{H}_0}_{\equiv T\mathcal{H}_0} \cdots \longrightarrow \mathcal{H}_0 \otimes \mathcal{H}_0 \otimes \mathcal{H}_0 \otimes \mathcal{H}_0 \cdots$$

$(Q, *, \mathcal{H}_0)$ is a DGA

$$(S,S) = 0 \iff \hat{m}^2 = [\hat{m}, \hat{m}] = 0 \quad \text{(plus cyclicity)}$$

graded commutator

$$S[\Psi] = \omega(\Psi, Q\Psi) + \frac{g}{3!}\,\omega(\Psi, \Psi * \Psi) + \overbrace{\omega(\Psi, m_3(\Psi,\Psi,\Psi)) + \cdots}^{\text{Deformation}}$$



$$\hat{m} = Q \otimes \mathbb{1} \otimes \mathbb{1} \cdots + m_2 \otimes \mathbb{1} \otimes \mathbb{1} \cdots + m_3 \otimes \mathbb{1} \otimes \mathbb{1} + \cdots$$

$$[\hat{m} + \delta\hat{m}, \hat{m} + \delta\hat{m}] = \underbrace{[\hat{m}, \hat{m}]}_{=0} + [\hat{m}, \delta\hat{m}] + \cdots \overset{B-V}{=} 0$$

→ Consistent def'n of OSFT is a cohomology problem

$d_H \equiv [\hat{m}, \cdot]$

- e.g. adding strips is a trivial, d_H-exct def'n

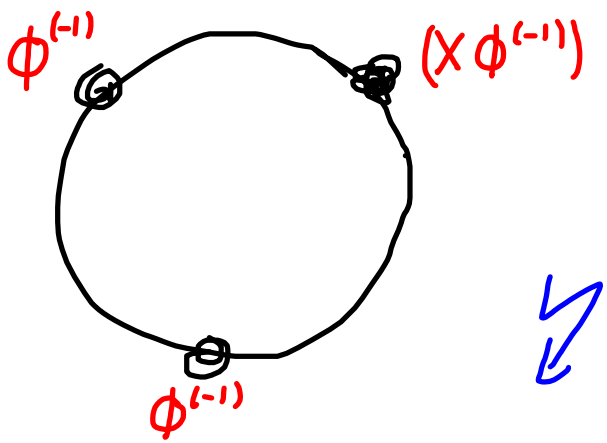
"Thm" (N. Moeller, I.S.) : $\mathrm{coh}(d_H) \cong \mathrm{coh}(Q_{closed})$

NS-sector: I am not aware of an analogue of a B-V equation for super moduli space. Usually one just assumes a decomposition that is mapped by a SCFT (matter + b, c + β, γ) to a B-V equation for $S[\phi, \bar{\phi}]$

$\xrightarrow{\text{integrate } \Theta}$ picture changing op. $X = Q\xi$

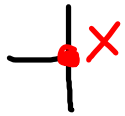
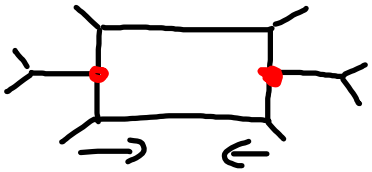
$$\boxed{\begin{aligned} &\gamma = \eta e^{\varphi}, \quad \beta = \partial\xi e^{-\varphi} \\ &V(z) = c\delta(\gamma) V_m = c e^{-\varphi} V_m \end{aligned}}$$

$(\xi \in \mathcal{H}_{large})$ Berkovits SFT

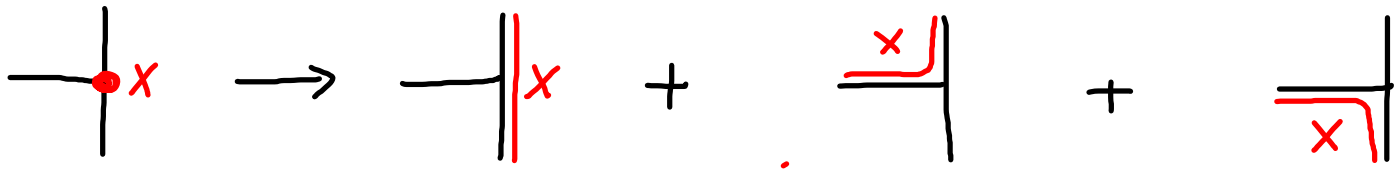


$\hat{=}$ integ'd vertex op.

$\Downarrow$ cyclicity

Witten '86 :  However, the 4-pt. fn  diverges

Alternative: (Erler, Konopka, I.S. '13)



$\chi(i) \longrightarrow \quad X = \int f(z)\, \chi(z)$ in local coord,

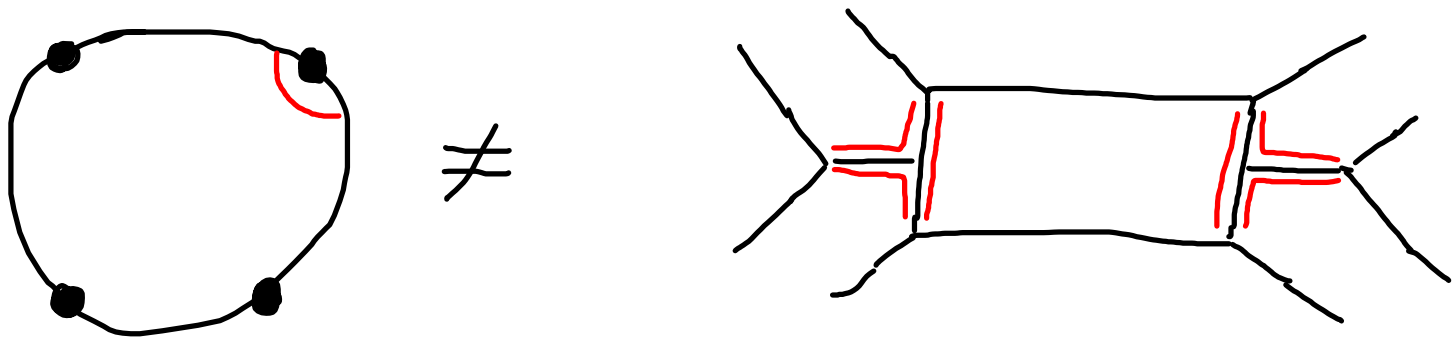
eg. $f(z) = \frac{1}{z} \quad \rightarrow \quad \chi \longleftrightarrow \chi_0$

in terms of maps:

$$M_2(\cdot,\cdot) = \frac{1}{3}\left(\chi\, m_2(\cdot,\cdot) + m_2(\chi\cdot,\cdot) + m_2(\cdot,\chi\cdot)\right)$$

Problems:

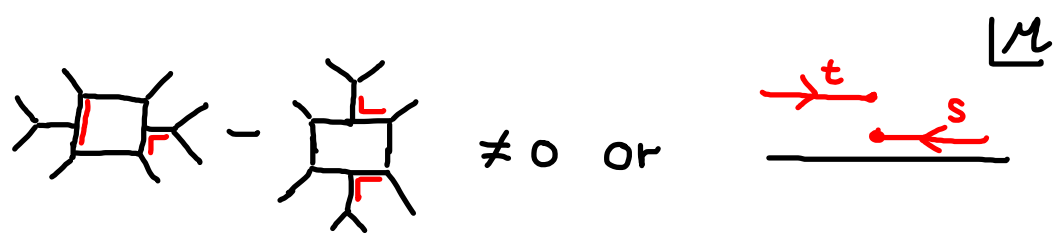
i) doesn't reproduce the perturb. 4-pt. amplitude:



ii) not gauge-invariant:

$$M_2(M_2(A,B),C) + (-1)^{|A|} M_2(A, M_2(B,C)) \neq 0$$

geometrically,



(see also Sen, Witten '15)

Both problems can be solved simultaneously by introducing a contact interaction M_3 s.t.

no bosonic moduli

$$M_2(M_2(A,B),C) + (-1)^{|A|} M_2(A, M_2(B,C))$$

$$+ Q M_3(A,B,C) + M_3(QA,B,C) + \text{cycl.} \overset{!}{=} 0$$

Can solve for M_3 by noticing that M_2 is Q-exact in the <u>large</u> Hilbert space:

$$M_2 = [Q, \mu_2] \,, \qquad \xi = \int f(z)\, \xi(z)$$

$$\mu_2(\cdot,\cdot) = \frac{1}{3}\left(\xi\, m_2(\cdot,\cdot) - m_2(\xi\,\cdot,\cdot) - (-1)^{|\cdot|} m_2(\cdot,\xi\,\cdot) \right)$$

then:

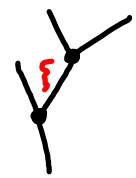
$$M_3 = \frac{1}{2}\left(M_2(\cdot, \mu_2(\cdot,\cdot) + \text{perm.} \right) + \frac{1}{2}[Q, \mu_3]$$

μ_3 is a priory arbitrary but can be fixed by imposing that M_3 preserves the small Hilbert space:

$$0 \overset{!}{=} [\eta, M_3]$$

$$= -\frac{1}{3}\left[(-1)^{|\cdot|} m_2(\cdot, \xi m_2(\cdot,\cdot)) + m_2(\times m_2(\cdot,\cdot), \cdot)\right] + \frac{1}{2}[Q,[\eta,\mu_3]]$$

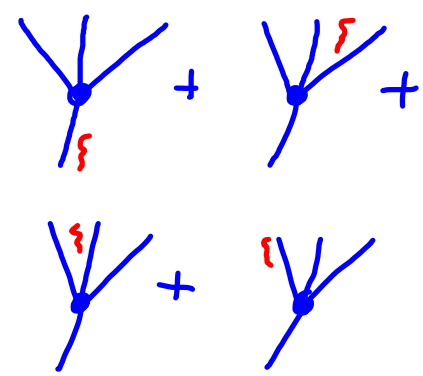
$$= Q\Big\{\underbrace{-\frac{1}{3}\left[(-1)^{|\cdot|} m_2(\cdot, \xi m_2(\cdot,\cdot)) + m_2(\xi m_2(\cdot,\cdot), \cdot)\right]}_{\equiv \frac{2}{3} m_3 \;=} + \frac{1}{2}[\eta,\mu_3]\Big\}$$



$m_2 = *$ associative

Now, since $[\eta, m_3] = [m_2, m_2] = 0$

$$\mu_3 = \frac{1}{4}\left[\xi\, m_3(\cdot,\cdot,\cdot) - m_3(\xi\cdot,\cdot,\cdot) + \text{perm.}\right]$$



guarantees that $[\eta, M_3] = 0$.

$$
\begin{tikzcd}
 & & M_4 \\
 & M_3 \arrow[u, "Q"'] & \mu_4 \arrow[u, "Q"'] \\
M_2 & \mu_3 \arrow[u, "\xi"'] \arrow[r] & m_4 \arrow[u, "\xi"'] \\
\mu_2 \arrow[u, "Q"'] \arrow[r] & m_3 \arrow[u, "\xi"'] & \\
m_2 \arrow[u, "\xi"'] & &
\end{tikzcd}
$$

Quartic vertex:

ω_L : symplectic form in $\mathcal{H}_L$

$M_2^{(0)} = *$

Recurrence relation:

$$M_{n+2} = \frac{1}{n+1} \sum_{k=0}^{n} [M_{k+1}, \mu_{n-k+2}]$$

$$M_1 = Q$$

Let:

$$\hat{M} = M_2 \otimes \mathbb{1} \otimes \mathbb{1} \cdots + M_3 \otimes \mathbb{1} \otimes \mathbb{1} \cdots + \cdots, \quad \hat{\mu} = \mu_2 \otimes \mathbb{1} \otimes \mathbb{1} + \cdots$$

then

$$\hat{M}_3 = \frac{1}{2}\left([\hat{M}_2, \hat{\mu}_2] + [\hat{Q}, \hat{\mu}_3]\right) = \frac{1}{2}[\hat{M}, \hat{\mu}]\Big|_3$$

Generating functions:

Let

$$\hat{M}(t) = \sum_{n=0}^{\infty} t^n \hat{M}_{n+1}$$

$$\hat{m}(t) = \sum_{n=0}^{\infty} t^n \hat{m}_{n+2}$$

$$\hat{\mu}(t) = \sum_{n=0}^{\infty} t^n \hat{\mu}_{n+2}$$

then:

$$\frac{d}{dt}\hat{M}(t) = [\hat{M}(t), \hat{\mu}(t)] \; ; \qquad \frac{d}{dt}\hat{m}(t) = [\hat{m}(t), \hat{\mu}(t)]$$

Ramond sector:

(T. Erler, S. Konopka, I.S. to appear)

$$\Psi_R = c e^{-\phi/2} |\text{matter}\rangle = \Psi_R^{(-1/2)}$$

Free theory is problematic: $S_0(\Psi_R) = \omega^{(2)}(\Psi_R, Q\Psi_R) \equiv 0$!

try $\omega(\Psi_R, \underset{\underset{X^{-1}}{\parallel}}{Y} Q\Psi_R)$, but X is degenerate off-shell $\Rightarrow$ Y ill defined

1) impose constraint on $\mathcal{H}_0$ s.t. Y exists

but, what about $\mathrm{coh}(Q) = \ker(Q)/Q(\cdots)$ ← reduced ?

2) can choose a gauge s.t. zero modes of X, Y are absent

but what about B-V ?

We will circumvent this problem by focussin on the e.m.

→ no quantization but
- tree-level amplitude
- space-time SUSY of class. sol'ns
- e.m. with R-R fields

<u>Ansatz</u>: $$0 = Q\phi_N + M_2(\phi_N, \phi_N) + m_2(\psi_R, \psi_R) + \cdots$$

$$0 = Q\psi_R + M_2(\phi_N, \psi_R) + M_2(\psi_R, \phi_N) + \cdots$$

more generally, if there are <u>less</u> than 2 Ramond inputs, we take M_n from the NS-sector. For 2 or more Ramond inputs we need new products:

eg. $[Q, M_3](R,R,R) + M_2 \circ m_2(R,R,R) \overset{\text{B-V}}{=} 0$

sol'n: $\tilde{M}_3(R,R,R) = -\mu_2 \circ m_2(R,R,R)$

then, $[Q, \tilde{M}_4](R,R,R,R) + \cancel{[\tilde{M}_3, m_2]}^{\,0}(R,R,R,R) = 0$

Thus, $\tilde{M}_4 \equiv 0$, no (R,R,R,R)-interaction in SFT !

The formulas already determine the general structure:

$$\mu_n\big|_0 \equiv \mu_n^{NS} \circ P_0$$ ← projects on $\emptyset$ or 1 Ramond input

$$M_n\big|_0 = M_n^{NS} \circ P_0$$

$$m_n\big|_0 = m_n^{NS} \circ P_0$$

projects on 2 or 3 R's

$$\tilde{M}_n\big|_2 = \tilde{M}_n \circ P_2$$

new

Recursion relation:

$$\tilde{M}_{n+3}\big|_2 = \frac{1}{n+1} \sum_{k=0}^{n} \left[\tilde{M}_{k+2}\big|_2 , \mu_{n-k+2}\big|_0 \right]$$

$$\begin{tikzcd}
 & M_2|_0 & & \tilde{M}_3|_2 & & M_3|_0 & & \tilde{M}_4|_2 & & \cdots \\
m_2|_0 \arrow[ur] & & \mu_2|_0 \arrow[ul] \arrow[ur] \arrow[dr] & & m_3|_0 \arrow[ur] & & \mu_3|_0 \arrow[ul] \arrow[ur] \arrow[dr] & & \mu_4|_0 \arrow[ur] & \\
 & & & & & & & m_4|_0 \arrow[ur] & &
\end{tikzcd}$$

equivalently:

$$M(t) = \sum_{n=0}^{\infty} t^n M_{n+1}\Big|_0$$

$$m(t) = \sum_{n=0}^{\infty} t^n m_{n+2}\Big|_0$$

$$\mu(t) = \sum_{n=0}^{\infty} t^n \mu_{n+2}\Big|_0$$

$$\tilde{M}(t) = \sum_{n=0}^{\infty} t^n \tilde{M}_{n+2}\Big|_2$$

then:

$$\frac{d}{dt} M(t) = [M(t), \mu(t)] \; ; \; \frac{d}{dt} \tilde{M}(t) = [\tilde{M}(t), \mu(t)]$$

$$\frac{d}{dt} m(t) = [m(t), \mu(t)] \; ; \; \mu(t) = \xi \cdot m(t)$$

Summary:

1) The NS sector of open superstring field theory is a "large" closed string (L_∞) gauge transformation of the free theory, $S = \omega(\Phi, Q\Phi)$.

2) NS+R OSSFT is given by the same (modulo projectors) of $Q + m_2$, $m_2 = *$ (Witten)

$\therefore$ The NS+R vertices are not cyclic (ie. do not come from an action). But the S-matrix derived from them is cyclic (S. Konopka, to appear)

3) This establishes the existence of a decomposition of Supermoduli space at the level of CFT.

4) type II and heterotic string works out similarly (Erler, Konopka, I.S. '14)