Monopole Walls, Quantum Field Theories in Five Dimensions, and Melting Crystals

We argue that the classical dynamics of doubly periodic monopoles is related to the moduli spaces of vacua of quantum field theories in five dimensions. By studying the moduli spaces of such monopoles we extract information about their asymptotic metrics and phases. Interesting combinatorics emerges. In particular, the asymptotic metrics are related to the volumes of melted crystals, while the phase structure is given in terms of fibered secondary polytopes and oriented matroids.