# *Deformations of exceptional field theory and the Romans mass.*

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Based on [1604.*****]
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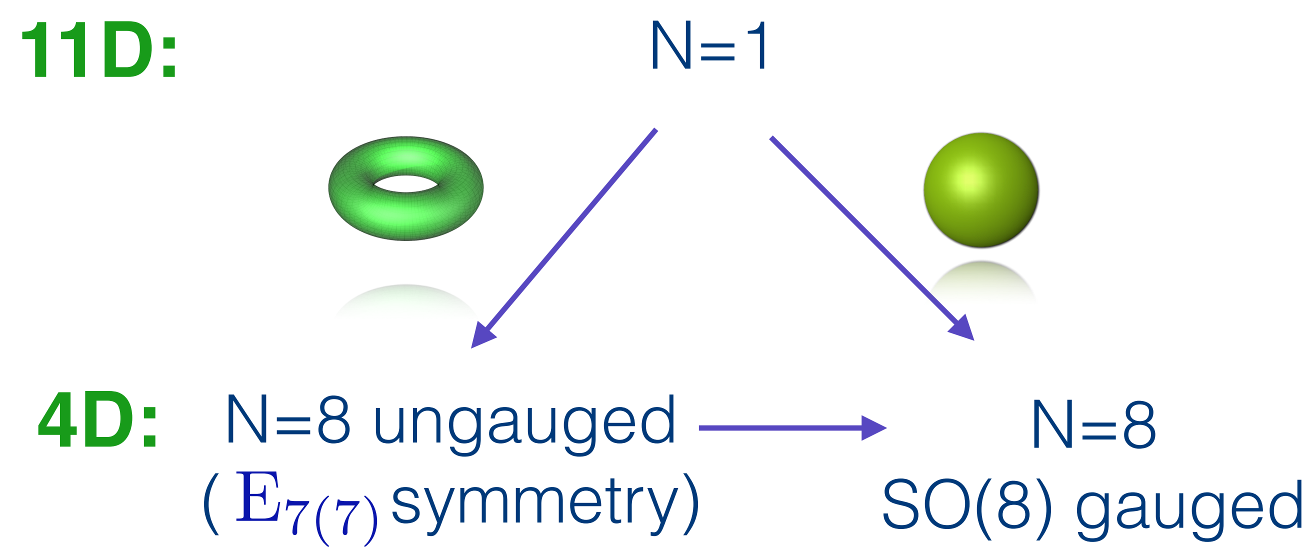
Maximal supergravities in various dimensions possess intriguing U-duality symmetries ($E_{7(7)}, E_{6(6)}, \ldots$).

Many of these theories can be obtained by dimensional reduction of 11D supergravity or 10D IIB supergravity, e.g.




> An old question is how the higher-dimensional supergravities reflect the exceptional symmetries which are present in their lower dimensional 'descendants'.

First approach: gauge equivalent rearrangement of degrees of freedom.

For example, 11D supergravity was rewritten from a 4D perspective while retaining all the degrees of freedom. **No truncation!**

*de Wit, Nicolai, 1980's*

→ These reformulations exhibit features of the U-dualities.

*New advances*: rely on extended geometrical structures and/or spurious degrees of freedom.

- Exceptional generalized geometry: **extension of the tangent space** to include the various p-forms associated with abelian gauge symmetries.

  → Unifies diffeomorphisms and p-forms gauge transformations in an enlarged symmetry group.

*Koepsell, Nicolai, Samtleben, 2000*
*Coimbra, Strickland-Constable, Waldram, 2011*
*Godazgar, Godazgar, Nicolai, 2013*
*etc…*

- Exceptional field theory (EFT): **extension of space-time** in order for the coordinates to transform covariantly under the exceptional duality symmetry.

*Hillman 2009*

  → Fully $\mathrm{E}_{n(n)}$-covariant theory.

  → A **section constraint** reduces the number of physical coordinates to (at most) 11.

*Berman, Godazgar, Perry 2011*
*Hohm, Samtleben, 2013*
*Samtleben, Musaev, 2014*
*etc…*

The $\mathrm{E}_{n(n)}$ EFT shares (most) of the field content and multiplet structure of gauged maximal supergravities in D=(11-n):

$$\left\{ e_\mu{}^\alpha,\ \underbrace{\mathcal{M}_{MN}}_{\text{Scalar matrix}},\ A_\mu{}^M,\ (2 \leq \text{p})\text{-forms} \right\}$$

$$\mu = 1, \ldots, (11-n)$$
$$M = 1, \ldots, \dim(\mathcal{R}_{\text{vec}})$$

All fields depend on an extended set of coordinates: $(x^\mu, Y^M)$ — **dual to** $\partial_M$

The EFT is uniquely determined by its **gauge symmetries**:

- Internal generalised diffeomorphisms generated by $\Lambda^M(x, Y)$
- External diffeomorphisms generated by $\xi^\mu(x, Y)$

$\mathrm{E}_{n(n)}$ EFT | $Y^M$

**D=11-n:** Ungauged maximal SUGRA | $x^\mu$

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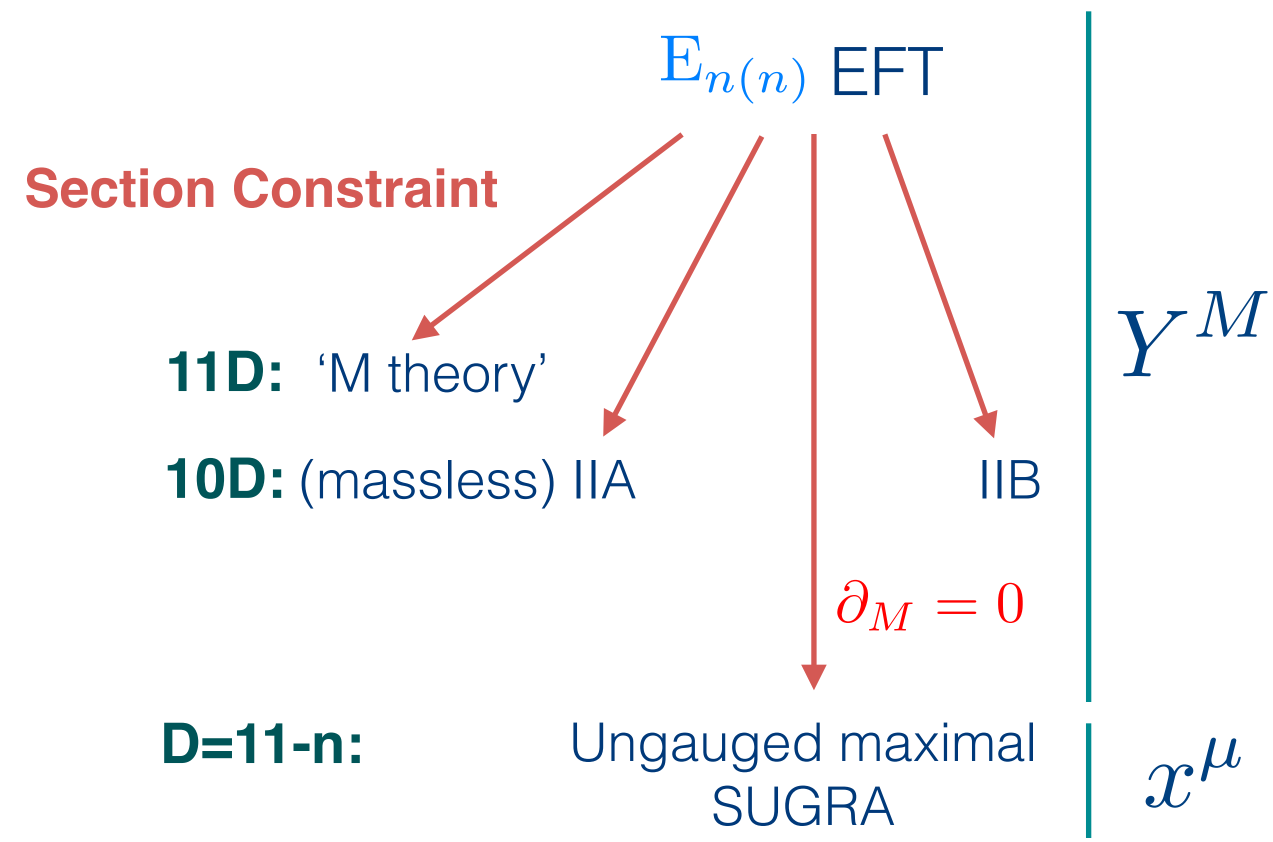
<u>Consistency of the internal diffeomorphisms</u> algebra requires a covariant section constraint.

→ **Recover physical theories** in a Kaluza-Klein split.



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**Scalar matrix**

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$\mathrm{E}_{n(n)}$ EFT

**Section Constraint**

**11D:** 'M theory'

**10D:** (massless) IIA | IIB

$\partial_M = 0$

**D=11-n:** Ungauged maximal SUGRA

$Y^M$

$x^\mu$

! Generalised diffeomorphisms **then** account for *both* the **internal** p-form gauge transformations and the ordinary **internal** diffeomorphisms.

Recently, *the manifest duality covariance* of EFT's was used to:

- compute gravitons scattering (on an extended torus):

  ➞ *Directly* obtain U-duality invariant expressions for the coefficients of higher-derivative terms in the M-theory effective action.

  *Bossard, Kleinschmidt, 2015*

- Prove the consistency of classes of truncations of eleven-dimensional and Type II supergravities:

Generalised Scherk-Schwarz

$$\mathcal{M}_{MN}(x,Y) = \underbrace{U_M{}^P(Y) U_N{}^Q(Y)}_{\text{Twist matrix} \in \mathrm{E}_{n(n)}} \mathcal{M}_{PQ}(x)$$

*Hohm, Samtleben, 2014*
*Malek, Samtleben, 2015*
*Baguet, Pope, Samtleben, 2015*

EFT's provide an **embedding** of the eleven-dimensional and **massless** Type II supergravities in a manifestly U-duality covariant framework.

***What about massive Type IIA supergravity?***

➞ Massless IIA and IIB are the **only** solutions of the section constraint.

➞ Usually requires some (modest) form of non-geometry. *Hohm, Kwak, 2011*

***(How) can it be embed fully geometrically in EFT?***

## Generalities of $\mathrm{E}_{n(n)}$ EFT $(2 \leq n < 8)$

Generalised diffeomorphisms act on covariant tensors (of weight $\lambda$) as a **generalised Lie derivative:**

Under $\mathbb{R}^+$

$$\boxed{\delta_\Lambda U^M = \mathbb{L}_\Lambda U^M}$$

$$\mathbb{L}_\Lambda U^M = \Lambda^N \partial_N U^M - U^N \partial_N \Lambda^M + \underbrace{Y^{MN}{}_{PQ}}_{\mathrm{E}_{n(n)}\ \text{invariant}} \partial_N \Lambda^P U^Q + (\lambda - \omega)\, \partial_P \Lambda^P U^M$$

Distinguished weight

- Two requirements for consistency of the theory:
  - Closure of the generalised diffeomorphisms according to an E-bracket:

$$\boxed{[\mathbb{L}_\Lambda, \mathbb{L}_\Sigma] = \mathbb{L}_{[\Lambda,\Sigma]_E}} \quad \text{with} \quad [\Lambda, \Sigma]_E^M \equiv \frac{1}{2}\left(\mathbb{L}_\Lambda \Sigma^M - \mathbb{L}_\Sigma \Lambda^M\right)$$

  - Jacobi identity: $\{\bullet, \bullet\}_E^M$ must be **trivial parameters.**

$$\boxed{\mathbb{L}_{\{\bullet,\bullet\}_E} = 0} \quad \text{with} \quad \{\Lambda, \Sigma\}_E^M \equiv \frac{1}{2}\left(\mathbb{L}_\Lambda \Sigma^M + \mathbb{L}_\Sigma \Lambda^M\right)$$

Require section constraint:

$$Y^{MN}{}_{PQ}\, \partial_M \otimes \partial_N = 0$$

- The vectors $A_\mu{}^M$ covariantize the external derivatives: $\mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu}$

$\longrightarrow$ They transform as: $\boxed{\delta_\Lambda A_\mu{}^M = \mathcal{D}_\mu \Lambda^M}$

⚠ Up to trivial gauge parameters

# Example (n=4): SL(5) EFT

Musaev 2015

U-duality in D=7 maximal SUGRA

$A, B = 1, \ldots, 5\,.$

- The theory contains vector fields ${A_\mu}^M \equiv {A_\mu}^{[AB]}$ in the **10** of SL(5).
  - ➔ It is defined on (7+10)-dimensional space coordinatized by $(x^\mu, Y^{AB})$

- The section constraint takes the form: $\boxed{\varepsilon^{ABCDE}\,\partial_{AB}\otimes\partial_{CD}=0}$

Upon the following branching: $\mathbf{5} \xrightarrow{\text{SL(4)}} \mathbf{4}+\mathbf{1} \xrightarrow{\text{SL(3)}} \mathbf{3}+\mathbf{1}+\mathbf{1}$

$$A \longrightarrow (a\,,\,4) \longrightarrow (\alpha\,,\,5\,,\,4)$$

The 10 vectors decompose as ${A_\mu}^{AB} = (\,{A_\mu}^{\alpha 5}\,,\,{A_\mu}^{\alpha 4}\,,\,{A_\mu}^{\alpha\beta}\,,\,{A_\mu}^{45}\,)$

The two SL(5)-orbits of solutions to the section constraint are:

1. **'M-theory':** $\partial_{a4}\neq 0\,,\,\partial_{ab}=0$ ⟶ **IIA:** $\partial_{\alpha 4}\neq 0\,,\,\partial_{45}=\partial_{\alpha 5}=\partial_{\alpha\beta}=0$
2. **IIB:** $\partial_{\alpha\beta}\neq 0\,,\,\partial_{45}=\partial_{\alpha 5}=\partial_{\alpha 4}=0$

Choosing the IIA solution, $\delta_\Lambda {A_\mu}^{AB} = (\partial_\mu + \mathbb{L}_\Lambda){A_\mu}^{AB}$ reproduces the internal gauge transformations and diffeomorphisms of type IIA in a 7+3 KK split.

# *7+3 split of massless IIA*

- Massless type IIA supergravity contains the p-forms: $\underbrace{A_m, A_{mnp}}_{\text{R-R}}, \underbrace{A_{mn}}_{\text{NS-NS}}$

- Under their associated gauge transformations they transform as:

$$\delta A_m = \partial_m \lambda \quad , \quad \delta A_{mn} = 2\,\partial_{[m}\,\Xi_{n]} \quad , \quad \delta A_{mnp} = 3\,\partial_{[m}\,\theta_{np]} - 3\,A_{[mn}\,\partial_{p]}\lambda$$

- We perform a 7+3 Kaluza-Klein split: $x^m = (x^\mu, y^\alpha)$

10D spacetime

7D spacetime

3D internal space

One obtains **10** 7D vectors. After standard KK redefinitions, they transform under internal diffeomorphisms and internal gauge transformations as:

KK vector

diffeo. parameter

$$\begin{aligned}
\delta B_\mu{}^\alpha &= (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\xi^\alpha + \xi^\delta\,\partial_\delta B_\mu{}^\alpha \ , \\
\delta C_\mu &= \xi^\delta\,\partial_\delta C_\mu + (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\lambda \ , \\
\delta C_{\mu\beta} &= \xi^\delta\,\partial_\delta C_{\mu\beta} + C_{\mu\delta}\,\partial_\beta \xi^\delta + (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\Xi_\beta + B_\mu{}^\delta\,\partial_\beta\,\Xi_\delta \ , \\
\delta C_{\mu\beta\gamma} &= \xi^\delta\,\partial_\delta C_{\mu\beta\gamma} + 2\,C_{\mu\delta[\gamma}\,\partial_{\beta]}\xi^\delta + (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\theta_{\beta\gamma} + 2\,B_\mu{}^\delta\,\partial_{[\beta|}\,\theta_{\delta|\gamma]} \\
&\quad + 2\,C_\mu\,\partial_{[\beta}\Xi_{\gamma]} - 2\,C_{\mu[\beta}\,\partial_{\gamma]}\lambda \ .
\end{aligned}$$

*All fields still depend on 10 coordinates!*

# 7+3 split of massive IIA

- The theory now contains a **deformation** (mass) **parameter**: $m_R$
- The gauge transformations of the p-forms are deformed:

$$\delta A_m = \partial_m \lambda + m_R \Xi_m \ , \ \delta A_{mn} = 2\,\partial_{[m}\,\Xi_{n]} \ , \ \delta A_{mnp} = 3\,\partial_{[m}\,\theta_{np]} - 3\,A_{[mn}\,\partial_{p]}\lambda - m_R\,A_{[mn}\Xi_{p]}$$

- After the 7+3 Kaluza-Klein split and the same KK redefinitions, the internal gauge transformations of the 10 7D vectors become:

$$\begin{aligned}
\delta B_\mu{}^\alpha &= (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\xi^\alpha + \xi^\delta\,\partial_\delta B_\mu{}^\alpha \\
\delta C_\mu &= \xi^\delta\,\partial_\delta C_\mu + (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\lambda - m_R\,B_\mu{}^\delta\,\Xi_\delta \\
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\delta C_{\mu\beta\gamma} &= \xi^\delta\,\partial_\delta C_{\mu\beta\gamma} + 2\,C_{\mu\delta[\gamma}\,\partial_{\beta]}\xi^\delta + (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\theta_{\beta\gamma} + 2\,B_\mu{}^\delta\,\partial_{[\beta|}\,\theta_{\delta|\gamma]} \\
&\quad +2\,C_\mu\,\partial_{[\beta}\Xi_{\gamma]} - 2\,C_{\mu[\beta}\,\partial_{\gamma]}\lambda - 2\,m_R\,C_{\mu[\beta}\,\Xi_{\gamma]}
\end{aligned}$$

The *new terms* ***cannot*** *be recovered from EFT since:* $\delta_\Lambda A_\mu{}^{AB} = (\partial_\mu + \mathbb{L}_\Lambda)A_\mu{}^{AB}$

→ ***Suggests a deformation of the generalised Lie derivative***

# Deformed generalised Lie derivative

- We define a modified Lie derivative by introducing a deformation parameter $X_M$ which takes values in the Lie algebra of the U-duality group:

$$\boxed{\widetilde{\mathbb{L}}_\Lambda = \mathbb{L}_\Lambda + \Lambda^M X_M \,, \qquad \text{with} \qquad X_M = \Theta_M{}^\alpha t_\alpha}$$

$\mathrm{E}_{n(n)}$ **generators**

<u>*Example*</u>: on a covariant vector U: $\delta_\Lambda U^N = \widetilde{\mathbb{L}}_\Lambda U^N = \mathbb{L}_\Lambda U^N - X_{MP}{}^N \Lambda^M U^P$

- In <u>gauged supergravities</u>, $\Theta_M{}^\alpha$ is the **embedding tensor**. It *selects* a subgroup of the U-duality group which is realised locally.

  It must satisfy two constraints**:**

  *de Wit, Samtleben, Trigiante, Nicolai, etc…*

  - *Quadratic constraint:*

    (closure of the gauge algebra): $\left[X_M, X_N\right] = X_{MN}{}^P X_P$

  - *Linear constraint:*

    Restricts $\Theta_M{}^\alpha$ to a specific representation:

    → Necessary for the tensor hierarchy

| $D$ | U-duality | $\mathcal{R}_\Theta$ |
|---|---|---|
| 7 | SL(5) | $\mathbf{15} + \mathbf{40}'$ |
| 4 | $\mathrm{E}_{7(7)}$ | $\mathbf{912}$ |

- We find that the *deformation parameter must satisfy these two constraints*.

# *Deformed EFT: 'XFT'*

Can we construct a deformed EFT from the deformed generalised Lie derivative? What are the (new) **constraints on the extended coordinates**?

As in EFT, consistency of the theory demands:

- Closure of the generalised diffeomorphisms according to an 'X-bracket':

$$\boxed{[\widetilde{\mathbb{L}}_\Lambda, \widetilde{\mathbb{L}}_\Sigma] = \widetilde{\mathbb{L}}_{[\Lambda,\Sigma]_X}} \quad \text{with } [\Lambda,\Sigma]_X^M \equiv \frac{1}{2}(\widetilde{\mathbb{L}}_\Lambda \Sigma^M - \widetilde{\mathbb{L}}_\Sigma \Lambda^M) = [\Lambda,\Sigma]_E^M - \Lambda^N \Sigma^P X_{[NP]}{}^M$$

- Triviality of the Jacobiator:

$$\boxed{\widetilde{\mathbb{L}}_{\{\bullet,\bullet\}_X} = 0} \quad \text{with } \{\Lambda,\Sigma\}_X^M \equiv \frac{1}{2}(\widetilde{\mathbb{L}}_\Lambda \Sigma^M + \widetilde{\mathbb{L}}_\Sigma \Lambda^M) = \{\Lambda,\Sigma\}_E^M - \Lambda^N \Sigma^P X_{(NP)}{}^M$$

This requires the following constraints on all the fields and the parameter $X_M$ :

***'XFT' constraints***

$$\begin{cases} Y^{MN}{}_{PQ}\, \partial_M \otimes \partial_N = 0 & \text{Section constraint of EFT} \\ X_{MN}{}^P \partial_P = 0 & \text{New constraint: } \underline{\text{'X-constraint'}} \\ [X_M, X_N] = X_{MN}{}^P X_P & \text{Quadratic constraint of gauged SUGRA} \end{cases}$$

***What are the solutions to these constraints?*** → ***SL(5) example***

# *SL(5) 'XFT' and the embedding of massive IIA*

Still defined on a 7+10 dimensional space $(x^\mu, Y^{[AB]})$ with $A, B = 1, \ldots, 5.$

**Linear constraint** $\longrightarrow$ The deformation parameter is the **15**+**40'** of SL(5):

$$X_{MN}{}^{P} \equiv X_{AB\,CD}{}^{EF} = 2\,X_{AB\,[C}{}^{[E}\delta^{F]}_{D]} = 2(\delta^{[E}_{[A}K_{B][C} - 2\,\varepsilon_{ABGH[C}Z^{GH,[E})\delta^{F]}_{D]}$$

$$\boxed{\text{We choose the deformation parameter to be the embedding tensor of the D=7 gauged SUGRA obtained upon truncation of massive IIA on a 3-torus.}}$$

$\longrightarrow$ **Quadratic constraint** trivially satisfied.

Branching SL(5) to SL(4) then to SL(3): $A \longrightarrow (a\,,\,4) \longrightarrow (\alpha\,,\,5\,,\,4)$

The deformation parameter reads: $\boxed{X_{\alpha\beta\,\gamma}{}^{5} = -2\,m_R\,\epsilon_{\alpha\beta\gamma}}$

**X-constraint:** $\longrightarrow$ **Section constraint:**

$m_R\,\partial_{\alpha 5} = m_R\,\underbrace{\partial_{45}}_{\text{'M-theory'}} = 0$ $\longrightarrow$ $\epsilon^{\alpha\beta\gamma}\,\partial_{\alpha 4}\,\partial_{\beta\gamma} = 0$

**Only SL(3) covariant!**

~~'M-theory': $\partial_{a4} \neq 0$~~

**D=10+7:** **SL(5) XFT**

$$\Big\downarrow \; \partial_{AB} = 0$$

$Y^{AB}$

**D=7:** **Gauged SUGRA**
with $X_M$ being the embedding tensor.

$x^{\mu}$

~~'M-theory': $\partial_{a4} \neq 0$~~

<u>Four</u> **SL(3)**-orbits of solutions:

**1. Massive IIA:**

$\partial_{\alpha 4} \neq 0$ ('Natural' IIA frame)

$X_M$ is the Romans mass.

**2. IIB:** $(\beta \neq \gamma)$

$\partial_{\beta 4}\, ,\, \partial_{\gamma 4}\, ,\, \partial_{\beta\gamma} \neq 0$

$X_M$ is a $F_{(1)}$ background flux.

**3. Massless IIA:** $(\alpha \neq \beta \neq \gamma)$

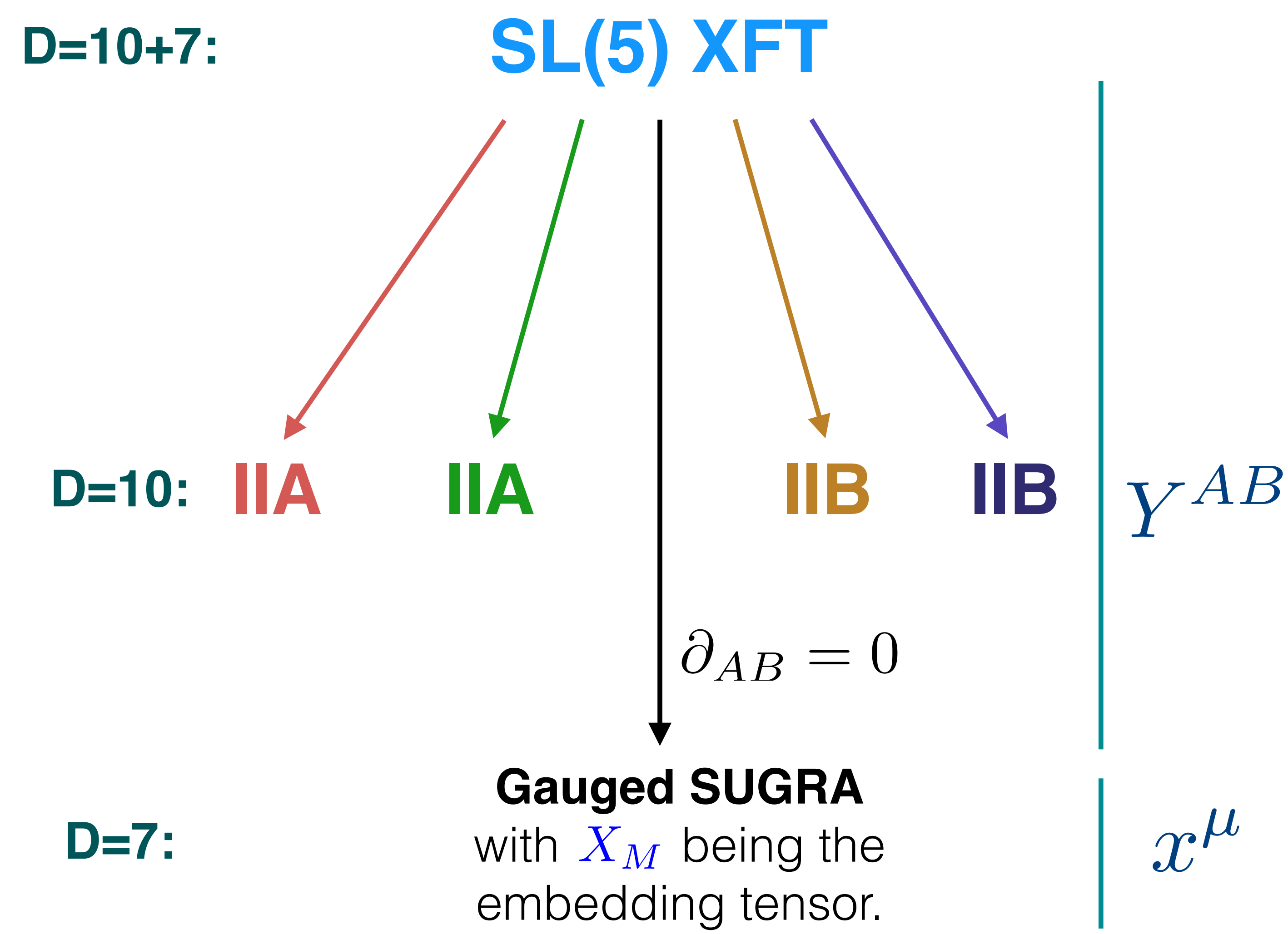
$\partial_{\gamma 4}\, ,\, \partial_{\alpha\gamma}\, ,\, \partial_{\beta\gamma} \neq 0$

$X_M$ is a $F_{(2)}$ background flux.

**4. IIB:**

$\partial_{\alpha\beta} \neq 0$ ('Natural' IIB frame)

$X_M$ is a $F_{(3)}$ background flux.




**1.** $\underset{\text{T-duality along } Y^{\alpha 4}}{\longrightarrow}$ **2.** $\underset{\text{T-duality along } Y^{\beta 4}}{\longrightarrow}$ **3.** $\underset{\text{T-duality along } Y^{\gamma 4}}{\longrightarrow}$ **4.**

~~‘M-theory’: $\partial_{a4} \neq 0$~~

<u>Four</u> **SL(3)**-orbits of solutions:

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$\partial_{\beta 4}\,,\ \partial_{\gamma 4}\,,\ \partial_{\beta\gamma} \neq 0$

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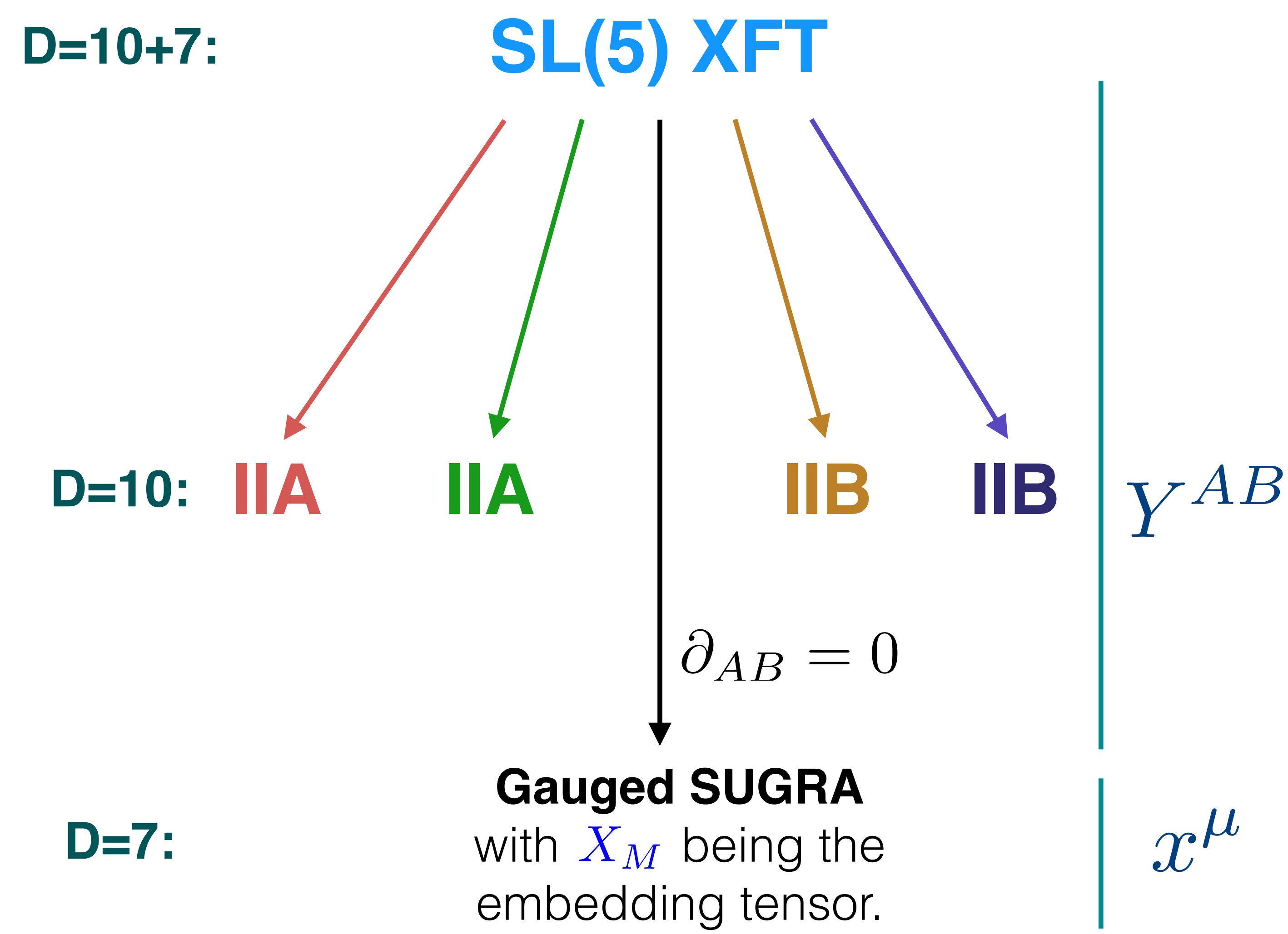


**1.** $\xrightarrow{}$ **2.** $\xrightarrow{}$ **3.** $\xrightarrow{}$ **4.**

T-duality along $Y^{\alpha 4}$ — T-duality along $Y^{\beta 4}$ — T-duality along $Y^{\gamma 4}$

After choosing **1.**, the theory reduces to massive IIA in a 7+3 Kaluza-Klein split.

In the (natural) **IIA frame** $\partial_{\alpha 4} \neq 0$ , the generalised diffeomorphisms:

$$\delta_\Lambda A_\mu{}^{AB} = (\partial_\mu + \widetilde{\mathbb{L}}_\Lambda) A_\mu{}^{AB} = (\partial_\mu + \mathbb{L}_\Lambda) A_\mu{}^{AB} - \Lambda^{CD} A_\mu{}^{EF} X_{CD\,EF}{}^{AB}$$

**reproduce precisely** the internal diffeomorphisms and internal gauge transformations of the 10 vectors of massive IIA in a 7+3 Kaluza-Klein split:

$$\begin{aligned}
\delta B_\mu{}^\alpha &= (\partial_\mu - B_\mu{}^\delta\,\partial_\delta)\,\xi^\alpha + \xi^\delta\,\partial_\delta B_\mu{}^\alpha \\
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\end{aligned}$$

***The 'XFT' extension of EFT also allows for the embedding of massive type IIA supergravity.***

**What about the dynamics of 'XFT'?**

# *Dynamics of 'XFT'*

In EFT, the **dynamics of the different fields can be encoded in an action** $\longrightarrow$ First requires to construct the **tensor hierarchy**.

The action is uniquely determined by requiring gauge invariance under external and internal generalised diffeomorphisms.

$\longrightarrow$ Following the EFT procedure, we construct the tensor hierarchy and the action for 'XFT'. We focus on the differences with EFT.

We leave the SL(5) 'XFT' behind and choose to construct the action for the $\mathrm{E}_{7(7)}$ 'XFT':

**U-duality in D=4 maximal SUGRA**

$\mathrm{E}_{7(7)}$ **EFT by:** *Hohm, Samtleben, 2013*

- <u>One of the most subtle case</u>: it should work for $n < 7$ .
- <u>Convenient</u>: the tensor hierarchy is short. Only a few p-forms are needed.

# $\mathrm{E}_{7(7)}$ ***'XFT': generalities***

- The vector fields $A_\mu{}^M$ are in the fundamental of $\mathrm{E}_{7(7)}$, $M = 1, \ldots, 56$ .

  ⟶ Theory defined on a (4+56)-dimensional space $(x^\mu, Y^M)$

- **Linear constraint** ⟶ The deformation parameter is in the **912** of $\mathrm{E}_{7(7)}$ :

  **No trombone gauging** | $X_{MN}{}^M \quad , \quad X_{(MNP)} = X_{MN}{}^Q \Omega_{PQ} = 0$ **Invariant symplectic form of** $\mathrm{E}_{7(7)}$

- The constraints on the coordinates are:

  **Section constraint:** $\Omega^{MN} \partial_M \partial_N = 0$ **X-constraint:** $X_{MN}{}^P \partial_P = 0$

  $\mathrm{E}_{7(7)}$ **generators** $\alpha = 1, \ldots, 133$ $(t^\alpha)^{MN} \partial_M \partial_N = 0$

# $\mathrm{E}_{7(7)}$ *'XFT': generalities*

- The vector fields $A_\mu{}^M$ are in the fundamental of $\mathrm{E}_{7(7)}$ , $M = 1, \ldots, 56$ .

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$\mathrm{E}_{7(7)}$ **generators** $\alpha = 1, \ldots, 133$ $(t^\alpha)^{MN}\partial_M\partial_N = 0$

The symmetric X-bracket $\{\bullet, \bullet\}_X$ splits into two trivial parameters of the form:

$$\Lambda^M = (t^\alpha)^{MN}\partial_N\chi_\alpha + \tfrac{1}{6} Z^{M,\alpha}\chi_\alpha$$

where $Z^{M,\alpha} = -X_{PQ}{}^M (t^\alpha)^{PQ}$ is the **intertwining tensor** in D=4 gauged SUGRA.

$$\Lambda^M = \Omega^{MN}\chi_N$$

**Covariantly constrained**

$$\mathbb{P}_{\mathbf{1+133}}(\chi_M \otimes \partial_N) = 0$$

$$\mathbb{P}_{\mathbf{1+133}}(\chi_M \otimes \chi_N) = 0$$

$$Z^{M,\alpha}\chi_M = 0$$

**New in 'XFT'**

# $\mathrm{E}_{7(7)}$ *'XFT': tensor hierarchy*

As in EFT, a naive expression for the field strength ${F_{\mu\nu}}^M$ does *not* transform covariantly:

$$
{F_{\mu\nu}}^M = 2\,\partial_{[\mu} {A_{\nu]}}^M - \big[A_\mu, A_\nu\big]_X^M \longrightarrow \delta_\Lambda {F_{\mu\nu}}^M \neq \widetilde{\mathbb{L}}_\Lambda {F_{\mu\nu}}^M
$$

The non-covariant terms are cancelled against the gauge variations of a set of **two-forms** $B_{\mu\nu\,\alpha}\,,\ B_{\mu\nu\,M}$ $\longrightarrow$ Beginning of a tensor hierarchy.

$$
{\mathcal{F}_{\mu\nu}}^M = {F_{\mu\nu}}^M \underbrace{- 12\,(t^\alpha)^{MN}\partial_N B_{\mu\nu\,\alpha} - 2\,Z^{M,\alpha} B_{\mu\nu\,\alpha}} \underbrace{- \tfrac{1}{2}\,\Omega^{MN} B_{\mu\nu\,N}}
$$

**Trivial gauge parameters** $\longrightarrow$ $\big[\mathcal{D}_\mu, \mathcal{D}_\nu\big] = -\widetilde{\mathbb{L}}_{F_{\mu\nu}} = -\widetilde{\mathbb{L}}_{\mathcal{F}_{\mu\nu}}$

Two-forms also carry their own gauge transformations with parameters $\Xi_\mu$ .

In particular: $\delta_\Xi {A_\mu}^M = 12(t^\alpha)^{MN}\partial_N \Xi_{\mu\,\alpha} + 2\,Z^{M,\alpha}\,\Xi_{\mu\,\alpha} + \tfrac{1}{2}\,\Omega^{MN}\Xi_{\mu\,N}$

Under **vector** (i.e. generalised diffeomorphisms) and **tensor gauge transformations**:

$$
\boxed{\delta_\Lambda {\mathcal{F}_{\mu\nu}}^M = \widetilde{\mathbb{L}}_\Lambda {\mathcal{F}_{\mu\nu}}^M \qquad \text{and} \qquad \delta_\Xi {\mathcal{F}_{\mu\nu}}^M = 0}
$$

# $\mathrm{E}_{7(7)}$ *'XFT': tensor hierarchy*

As in EFT, a naive expression for the field strength $F_{\mu\nu}{}^M$ does *not* transform covariantly:

$$F_{\mu\nu}{}^M = 2\,\partial_{[\mu}A_{\nu]}{}^M - [A_\mu, A_\nu]^M_X \longrightarrow \delta_\Lambda F_{\mu\nu}{}^M \neq \widetilde{\mathbb{L}}_\Lambda F_{\mu\nu}{}^M$$

The non-covariant terms are cancelled against the gauge variations of a set of **two-forms** $B_{\mu\nu\,\alpha}\,,\ B_{\mu\nu\,M}$ $\longrightarrow$ Beginning of a tensor hierarchy.

$$\mathcal{F}_{\mu\nu}{}^M = F_{\mu\nu}{}^M - \underline{12\,(t^\alpha)^{MN}\partial_N B_{\mu\nu\,\alpha} - \boxed{2\,Z^{M,\alpha}B_{\mu\nu\,\alpha}}} - \underline{\tfrac{1}{2}\,\Omega^{MN}B_{\mu\nu\,N}}$$

**Trivial gauge parameters** $\longrightarrow$ $[\mathcal{D}_\mu, \mathcal{D}_\nu] = -\widetilde{\mathbb{L}}_{F_{\mu\nu}} = -\widetilde{\mathbb{L}}_{\mathcal{F}_{\mu\nu}}$

Two-forms also carry their own gauge transformations with parameters $\Xi_\mu$ .

In particular: $\delta_\Xi A_\mu{}^M = 12(t^\alpha)^{MN}\partial_N \Xi_{\mu\,\alpha} + \boxed{2\,Z^{M,\alpha}\,\Xi_{\mu\,\alpha}} + \tfrac{1}{2}\,\Omega^{MN}\Xi_{\mu\,N}$

Under **vector** (i.e. generalised diffeomorphisms) and **tensor gauge transformations**:

$$\boxed{\delta_\Lambda \mathcal{F}_{\mu\nu}{}^M = \widetilde{\mathbb{L}}_\Lambda \mathcal{F}_{\mu\nu}{}^M \qquad \text{and} \qquad \delta_\Xi \mathcal{F}_{\mu\nu}{}^M = 0}$$

These results are *consistent* with the $\mathrm{E}_{7(7)}$ EFT (when $X_M = 0$) and with D=4 gauged supergravities (when $\partial_M = 0$).

# $\mathrm{E}_{7(7)}$ *'XFT': the bosonic action*

Same field content as for the $\mathrm{E}_{7(7)}$ EFT: $\{e_\mu{}^\alpha, \underbrace{\mathcal{M}_{MN}}_{\text{parametrising } \mathrm{E}_{7(7)}/\mathrm{SU}(8)}, A_\mu{}^M, B_{\mu\nu\,\alpha}, \underbrace{B_{\mu\nu\,M}}_{\text{Decouples when } \partial_M = 0}\}$

The **field equations** of 'XFT' can be encoded into a gauge invariant action supplemented by a twisted self-duality equation:

$$\boxed{\mathcal{F}_{\mu\nu}{}^M = -\tfrac{1}{2}\, e\, \varepsilon_{\mu\nu\rho\sigma}\, \Omega^{MN}\, \mathcal{M}_{NK} \mathcal{F}_{\rho\sigma}{}^K}$$

$$S_{\text{XFT}} = \int d^4x\, d^{56}Y\, e \Big[ \hat{R} + \frac{1}{48} g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{MN}\, \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{8} \mathcal{M}_{MN}\, \mathcal{F}^{\mu\nu M} \mathcal{F}_{\mu\nu}{}^N + e^{-1} \mathcal{L}_{\text{top}} - V_{\text{XFT}}(\mathcal{M}, g, X) \Big]$$

- Each term is invariant under *generalised diffeomorphisms*
- Relative coefficients are fixed by *external diffeomorphisms*

**Uniquely determines** $S_{\text{XFT}}$

The action takes the same form as in EFT, but with modified expressions for $\mathcal{D}_\mu$ and $\mathcal{F}_{\mu\nu}$ required by gauge covariance. ***Except for the potential***

# $\mathrm{E}_{7(7)}$ *'XFT': the potential*

The **scalar potential of 'XFT'** splits into three parts:

$$V_{\text{XFT}}(\mathcal{M}, g, X) = \underline{V_{\text{EFT}}(\mathcal{M}, g)} + \underline{V_{\text{SUGRA}}(\mathcal{M}, X)} + \underline{V_{\text{cross}}(\mathcal{M}, X)}$$

with

$$\begin{aligned} V_{\text{EFT}} = & -\tfrac{1}{48}\,\mathcal{M}^{MN}\,\partial_M\mathcal{M}^{KL}\,\partial_N\mathcal{M}_{KL} + \tfrac{1}{2}\,\mathcal{M}^{MN}\,\partial_M\mathcal{M}^{KL}\,\partial_L\mathcal{M}_{NK} \\ & -\tfrac{1}{2}\,g^{-1}\partial_M g\,\partial_N\mathcal{M}^{MN} - \tfrac{1}{4}\,\mathcal{M}^{MN}\,g^{-1}\partial_M g\,g^{-1}\partial_N g \\ & -\tfrac{1}{4}\,\mathcal{M}^{MN}\,\partial_M g^{\mu\nu}\,\partial_N g_{\mu\nu} \end{aligned}$$

**Vanishes when**

$\partial_M = 0$

$$V_{\text{SUGRA}} = \tfrac{1}{168}\left[X_{MN}{}^{P}X_{QR}{}^{S}\mathcal{M}^{MQ}\mathcal{M}^{NR}\mathcal{M}_{PS} + 7X_{MN}{}^{P}X_{QP}{}^{N}\mathcal{M}^{MQ}\right]$$

**Vanishes when**

$X_M = 0$

$$\boxed{V_{\text{cross}} = \tfrac{1}{12}\,\mathcal{M}^{MN}\mathcal{M}^{KL}X_{MK}{}^{R}\,\partial_N\mathcal{M}_{RL}}$$

**Vanishes when**

$\partial_M = 0$ **and/or** $X_M = 0$ ← **Exclusive to XFT**

The action reduces to the one of EFT (when $X_M = 0$ ) and to the one of D=4 gauged SUGRA (when $\partial_M = 0$ ).

Finally, the action reduces to the one of massive IIA in a 4+6 KK split upon choosing the correct deformation and set of physical coordinates.

## *Conclusion and outlook*

- We extend the EFT framework by introducing a consistent deformation of the generalised Lie derivative.

- Consistency of the generalised diffeomorphisms algebra imposes an additional constraint on the coordinate dependence of the fields.

- For a specific choice of deformation, the 'XFT' constraints admit 10D solution corresponding to massive IIA supergravity.

- The $\mathrm{E}_{7(7)}$ 'XFT' action takes a similar form as in EFT except for the scalar potential which contains a non-trivial extension.

**(Some) future directions:**

- Study truncations of massive IIA supergravity via generalised Scherk-Schwarz.

*Guarino, Jafferis, Varela, 2015*

- Implementation of D8 branes in an exceptional framework.

**Thank you for your attention.**

# Backup

# *SL(5) XFT: generic deformations*

For a **specific *frame*** (i.e.choice of coordinates), what are **all the deformations compatible with the XFT constraints**?

Reminder: $X_{MN}{}^{P} \equiv X_{AB\,CD}{}^{EF} \to \underbrace{K_{(AB)}}_{\mathbf{15}} \oplus \underbrace{Z^{[AB],C}}_{\mathbf{40'}}$

- 'M-theory' frame: $\partial_{a4} \neq 0$
  The only *allowed* deformation is identified with a Freund-Rubin parameter:

  $$K_{44} \equiv f_{\mathrm{FR}} \quad \text{4-form flux}$$

- (natural) IIA frame: $\partial_{\alpha 4} \neq 0$
  Allowed deformations correspond to background fluxes + *Romans mass:*

$$\tfrac{1}{4}\, K_{\alpha 4} = \tfrac{1}{2}\, \epsilon_{\alpha\beta\gamma}\, Z^{\beta\gamma,5} \equiv H_{\alpha} \quad \text{Dilaton flux}$$

$$K_{44} \equiv \tfrac{1}{3!}\, \epsilon^{\alpha\beta\gamma} H_{\alpha\beta\gamma} \quad \text{NS-NS 3-form flux}$$

$$Z^{5\alpha,5} \equiv \tfrac{1}{2}\, \epsilon^{\alpha\beta\gamma} F_{\beta\gamma} \quad \text{R-R 2-form flux}$$

$$Z^{45,5} \equiv \tfrac{1}{2}\, m_R$$

Cannot be all turned on simultaneously: restrictions from the **quadratic constraint**.

$$m_R\, H_{\alpha} = 0$$

$$H_{[\alpha}\, F_{\beta\gamma]} + \tfrac{1}{72}\, m_R\, H_{\alpha\beta\gamma} = 0$$

→ Tadpole cancellation for O8/D8 and O6/D6

- (natural) IIB frame: $\partial_{\alpha\beta} \neq 0$
  Allowed deformations also correspond to background fluxes + ?:

$$Z^{45,4} \equiv \frac{1}{3!}\,\epsilon_{\alpha\beta\gamma} H^{\alpha\beta\gamma}$$ NS-NS 3-form flux

$$Z^{45,5} \equiv \frac{1}{3!}\,\epsilon_{\alpha\beta\gamma} F^{\alpha\beta\gamma}$$ R-R 3-form flux

**SL(2) doublet of deformations**

$$Z^{\alpha 5,4} = Z^{\alpha 4,5} \equiv H^{\alpha}$$ Dilaton flux

$$Z^{5\alpha,5} \equiv F^{\alpha}$$ Axion flux

$$Z^{\alpha 4,4} \equiv ?$$

**SL(2) triplet of deformations**

Restrictions from the **quadratic constraint**.

$$\epsilon_{\alpha\beta\gamma}\, F^{\beta}\, H^{\gamma} = 0$$

$$\varepsilon_{\alpha\beta\gamma} F^{\beta}\, ? = 0$$

$$\varepsilon_{\alpha\beta\gamma} H^{\beta}\, ? = 0$$

The identification of one of the allowed deformation remains mysterious:

$\longrightarrow$ (non-geometric) one-form background flux: ***Under investigation***.

*Aldazabal, Andres, Camara, Grana 2013*

**The topological term**: can be written as manifestly gauge invariant **surface term** in a five-dimensional external space.

$$S_{\text{top}}(X) = -\tfrac{1}{24}\int_{\Sigma^5} d^5x \int d^{56}Y\, \varepsilon^{\mu\nu\rho\sigma\tau}\, \mathcal{F}_{\mu\nu}{}^{M}\, \mathcal{D}_\rho \mathcal{F}_{\sigma\tau\, M}$$
$$\equiv \int_{\partial\Sigma^5} d^5x \int d^{56}Y\, \mathcal{L}_{\text{top}}(X)$$

→ Contrarily to EFT, it does not vanish when $\partial_M = 0$. This was expected from the D=4 gauged supergravity action.