Duality for spin-2 (and higher-spin) fields around flat and AdS_D spacetimes

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based on [1512.09060] in collaboration with Thomas Basile and Xavier Bekaert,

and further developpments soon to appear with Andrea Campoleoni.

Also based on review of results with Paul Cook and D. Ponomarev

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PLAN OF TALK

0. Motivations

1. What kind of duality here?

2. Dualities for free spin-2 fields in flat spacetime: review of (old) results
   and not so old results: higher-dualizations

3. Spin-2 dual in AdS_D

⓪ Motivations

Electromagnetism

- $\vec{\nabla}\cdot\vec{E} = -\rho_e$, $\vec{\nabla}\cdot\vec{B} = -\rho_m$, $\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{k}$, $\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = -\vec{j}$

$\hookrightarrow$ Invariant under $\begin{pmatrix}\vec{E}\\ \vec{B}\end{pmatrix} \longrightarrow \begin{pmatrix}0 & 1\\ -1/c^2 & 0\end{pmatrix}\begin{pmatrix}\vec{E}\\ \vec{B}\end{pmatrix}$

if $K^\mu = (\rho_m, \vec{k}) = 0 = K^\mu = (\rho_e, \vec{j})$

There is more ... Maxwell again, but use diff. forms (and $c=1$) $F_{[2]} = d\,A_{[1]}$

$$d\begin{pmatrix}F\\ {}_*F\end{pmatrix} = \begin{pmatrix}{}_*K\\ {}_*J\end{pmatrix} \rightsquigarrow \begin{cases} 3\,\partial_{[\mu}F_{\nu\rho]} = \sqrt{-g}\,\varepsilon_{\mu\nu\rho\sigma}K^\sigma \\ \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}\,F^{\alpha\beta}\right) = J^\beta \end{cases}$$

Magnetic & electric charges

$$g_m = \int_V *K_{[1]} = \oint_{\partial V} F_{[2]} \quad , \qquad q = \int_V *J_{[1]} = \oint_{\partial V} *F_{[2]}$$

$$\begin{pmatrix} F \\ *F \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} F \\ *F \end{pmatrix}$$ SO(2) symmetry .

In case of currents $$\begin{pmatrix} K \\ J \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} K \\ J \end{pmatrix}$$

in particular

$$\boxed{\begin{matrix} g_m \longrightarrow -q \\ q \longrightarrow g_m \end{matrix} \qquad \begin{pmatrix} B \\ E \end{pmatrix} \longrightarrow \begin{pmatrix} -E \\ B \end{pmatrix}}$$

- Dualities of various kinds (S , T , U , ...)

  ↳ better understanding and discovery of fundamental dynamical systems. Open door to nonperturbative sectors (e.g. magnetic monopoles)

e.g. Cremmer, Julia, Scherk 11D SUGRA [1978]

$$\downarrow \text{Torus } T^7$$

Cremmer, Julia [1978] $\mathcal{N}_4 = 8$ sugra ungauged

↳ $\exists$ rigid $E_{7(7)}$ rotates $A_\mu^{M=1,\dots,56}$ electric & magnetic vectors

$E_{7(7)}$ mixes Field equations with Bianchi identities

The 3-form $A_{[3]} = \frac{1}{3!} dx^{M_1} dx^{M_2} dx^{M_3} A_{M[3]}(x)$ of 11D sugra

$\rightsquigarrow$ minimal-couples to M_2 "electric" brane

$\rightsquigarrow$ $\exists$ "magnetic" M_5 couples to $\tilde{A}_{[6]}$ form.

$E_{7(7)}(\mathbb{Z})$ knows about the dual $\tilde{A}_{[6]}$ and strong-weak dualities in strings IIA IIB 10D models.

Linearized gravity

$\hookrightarrow$ Hope to include spin-2 in the picture!

Various motivations, some of them from the e_{10} & e_{11} programs

[Julia, West, Damour-Henneaux-Nicolai, de Wit, Bergshoeff, Obers-Pioline, Bossard-Kleinschmidt, Hohm-Samtleben, Cook, Sezgin, Dall'Agata, Trigiante & many others]

① What kind of duality here? ⇝ off-shell

↳ Through a parent action :

- $S^{father}[A_{[p]}, H_{[n-p-1]}] = \frac{1}{2}\int_{M_n} (H \wedge * H - dA \wedge H)$

$$\hookrightarrow Z = \int \mathcal{D}A\, \mathcal{D}H \; exp\left(i\, S^{father}[A, H]\right)$$

Ⓐ Complete the square : $S^{fath.}[A,H] = \frac{1}{2}\int_{M_n} \Big[\underbrace{(H + * dA)}_{\tilde{H}} * (H + * dA) + dA \wedge * dA \Big]$

& $\int \mathcal{D}\tilde{H}$

$$\hookrightarrow Z \propto \int \mathcal{D}A \; exp \frac{i}{\hbar}\left[\frac{1}{2}\int (dA_{[p]} \wedge * dA_{[p]})\right]$$

$$\hookrightarrow \boxed{S^{daughter}[A_{[p]}] = \frac{1}{2}\int_{M_n} dA \wedge * dA = \frac{-1}{2(p+1)!}\int d^n x\, F^{\mu[p+1]} F_{\mu[p+1]}}$$

where $F_{[p+1]} := dA_{[p]}$

(B) Integrate $\int \mathcal{D}A$ in Z, using $\int \mathcal{D}A \, \exp\left[\frac{i}{\hbar} \int -\frac{1}{2} dA \wedge H\right] \sim \delta(dH)$

↳ effectively replace $H_{[n-p-1]}$ by $dB_{[n-p-2]}$

inside $Z = \int \mathcal{D}A \, \mathcal{D}H \; e^{\frac{i}{\hbar} S^f[A,H]}$

$$\Rightarrow \quad Z \sim \int \mathcal{D}A \, \mathcal{D}H \; e^{\frac{i}{\hbar} S^{son}[B_{[n-p-2]}]}$$

where

$$\boxed{S^{son}[B_{[n-p-2]}] = \frac{1}{2} \int_{M_n} dB \wedge * dB = \frac{-1}{2(p+1)!} \int d^n x \, H^{\mu[n-p-1]} H_{\mu[n-p-1]}}$$

where $H_{[n-p-1]} := dB_{[n-p-2]}$

↳ Note: Children (may) have 2 parents:

- $S^{Mother}[F_{[p+1]}, B_{[n-p-2]}] = \frac{1}{2} \int_{M_n} (F \wedge * F - dB \wedge F)$
- $S^{father}[A_{[p]}, H_{[n-p-1]}] = \frac{1}{2} \int_{M_n} (H \wedge * H - dA \wedge H)$

(2) Off-shell duality for massless gauge fields in flat space

[P. WEST 2001 & 2002, N.B., S. CNOCKAERT, M. HENNEAUX 2003]

In flat background $\Lambda = 0$

- $$S^{\text{fath.}}[e_{a|b}, Y^{ab|}{}_c] = -2\int d^n x \left(2 Y^{ab|c}\partial_a e_{b|c} - \tfrac{1}{2} Y_{ab|}{}^c Y^a{}_{c|}{}^b + \tfrac{1}{2(n-2)} Y_{ab|}{}^b Y^{ac|}{}_c \right)$$

(A) $0 \approx \dfrac{\delta S}{\delta Y}$ $\rightsquigarrow$ $$\boxed{S^{\text{daugh.}}[e_{a|b}] = -\int d^n x \left(\Omega^{ab|c}\Omega_{ab|c} + 2\,\Omega^{ab|c}\Omega_{ac|b} - 4\,\Omega^{ab|}{}_b \Omega_{ac|}{}^c \right)}$$

where $\Omega_{ab|c} := 2\,\partial_{[a} e_{b]|c}$

↳ That is Weyl's 1929 formulation of GR, linearized.

It enjoys local Lorentz: $\delta_L e_{a|b} = \epsilon_{ab}(x)$, $\epsilon_{ab} = -\epsilon_{ba}$.

(B) $S^{\text{son}}[\tilde{Y}_{abc|}{}^d]$ where $\tilde{Y}_{abc|}{}^d$ identically solves "$dH = 0$", i.e. $\partial_a Y^{ab|}{}_c = 0$

from $\int \mathcal{D}e$ in Z $\rightsquigarrow$ $$\boxed{S^{\text{son}}[\tilde{Y}_{abc|}{}^d] = \int d^n x \left(Y_{ab|}{}^c Y^a{}_{c|}{}^b - \tfrac{1}{n-2} Y_{ab|}{}^b Y^{ac|}{}_c \right)}$$

where $Y^{ab|}{}_d = \partial_c \tilde{Y}^{abc|}{}_d$.

Dualize : $\boxed{C_{\mu[n-3]|}{}^{a} := \frac{1}{3!}\,\epsilon_{\mu_1\cdots\mu_{n-3}\nu_1\nu_2\nu_3}\,\tilde{Y}^{\nu_1\nu_2\nu_3|a} \overset{\text{Not.}}{=} \frac{1}{6}\,\epsilon_{\mu[n-3]\nu[3]}\,\tilde{Y}^{\nu[3]|a}}$

Lorentz symmetry of $e_{a|b} \rightarrow \delta_L C_{\mu[n-3]|\alpha} = \epsilon_{\mu[n-3]\alpha\beta\gamma}\,\epsilon^{\beta\gamma}(x)$

Hence, from $C_{[n-3|1]} \sim Y_{GL(n)}[n-3,1] \oplus Y_{GL(n)}[n-2]$, one keeps 1^{st} term :

$$\hookrightarrow\ S^{son}[C_{[n-3|1]}] = S[T_{\mu[n-3]|\alpha}] = -\frac{1}{2(n-2)!}\int d^n x\ \mathcal{L}(T)$$

where

$$\boxed{\mathcal{L}(T) = -\frac{1}{(n-1)!}\,\epsilon^{e\mu[n-2]\alpha}\,\epsilon_{\nu\sigma[n-2]\alpha}\,\partial^{\sigma_1}T^{\sigma_2\cdots\sigma_{n-2}|}{}_{e}\,\partial_{\mu_1}T_{\mu_2\cdots\mu_{n-2}|}{}^{\nu}}$$

"Curtright" Lagrangian (the above form of $\mathcal{L}$ was given in

[X. Bekaert, N.B. & S. Cnockaert 0407102] for $Y_{GL_n}[p,q]$ gauge fields)

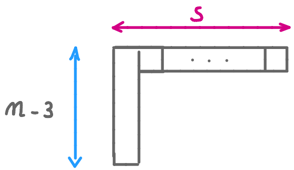
- **Note**: The other parent action is $S^{\text{Moth.}}[\Omega_{ab|c}, \tilde{Y}^{abc|}{}_{d}]$

↳ $$S^{\text{Moth.}} = -\int d^n x \left(2\,\Omega_{ab|c}\,\partial_d \tilde{Y}^{abd|c} + \Omega^{ab|c}\,\Omega_{ab|c} + 2\,\Omega^{ab|c}\,\Omega_{ac|b} - 4\,\Omega^{ab|}{}_{b}\,\Omega_{ac|}{}^{c} \right)$$

- In flat background, the dual graviton is

$$T_{\mu[n-3]|\nu} \sim$$  $$\sim Y_{GL_n}[n-3, 1]$$

- In flat background, the dual spin-s field is [N.B., S.CNOCKAERT, M.HENNEAUX 2003]

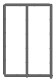
$T_s \sim$ 

where the equivalent of Weyl's 1929 linearized action is Vasiliev's 1981 quadratic action.

- The frame-like, 1st order action for T_s and the class of Labastida fields was given in [E. Skvortsov, 2008]

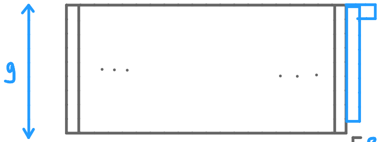
Spin-2 extensions ($\Lambda = 0$) : [N.B., Paul P. Cook, D. Ponomarev 2012]

- double-dual graviton $\quad S[D_{[n-3,n-3]}] \sim \int d^n x \; \partial D \, \partial D$

where $D \sim \{$  $\oplus$  $\oplus$  $\oplus \; \omega_{[n-6]} \}$

$[n-3 \,|\, n-3]$ $\qquad [n-2, n-4]$ $\qquad [n-1 \,|\, n-5]$

- As well as the "infinite dual tower" : (e.g. $n = 11$)



where extra fields necessarily enter the action.

These extra fields are contained in E_{11} and are associated with nul and imaginary roots of E_{11}

- This construction $\longrightarrow$ for any type of gauge field [N.B., D. Ponomarev 2012]

e.g. • $A_{[1]}$ and $A_{[3]}$: [N.B., P. Sundell, P. West 2015]

• $A_{[2]} \longrightarrow$ relation with DFT [Bergshoeff, Hohm, Penas, Riccioni 2016]

Summary of extensions in flat geometry for spin-2

$$
\begin{tikzcd}[column sep=small]
 & S^{Mo.}[\Omega_{ab|c}\,,\, Y_{a[3]|c}] \arrow[dr] & & S\,[H_{[n-3,2]}\,,\,\tilde{D}_{[3,n-3]}] \arrow[dr] & & \text{double-dual } \infty \text{ tower} \\
S^{Weyl}[e_{a,b}] \arrow[ur] & & S^{Curt.}[C_{[n-3,1]}] \arrow[ur] \arrow[dr] & & S^{DD}[D] \arrow[ur] & \\
 & & & S\,[G^{(1)}_{[1,n-3,1]}\,,\,\tilde{F}^{(1)}_{[2,n-3,1]}] \arrow[dl] & & \\
 & & S\,[F^{(1)}_{[n-2,n-3,1]}] \text{ and } \infty \text{ dual tower} & & &
\end{tikzcd}
$$

$$\mathcal{L}(C_{n-3,1})$$

though with no curl

$$S\,[G^{(1)}_{[1,n-3,1]}\,,\,\tilde{F}^{(1)}_{[2,n-3,1]}] \sim \int \left(d_1^{\dagger}\tilde{F}\cdot G + \overbrace{G\,G} \right)$$

$$G_{[1,n-3,1]} = d_1\, C_{[0,n-3,1]}$$

$$F^{(m)}_{[\underbrace{n-2,\,n-2,\,\dots,\,n-2}_{m \text{ columns}},\,n-3,\,1]}$$

③ Off-shell duality in AdS_D :

[1512.09060] with Th. Basile and X. Bekaert

Starts from linearized MacDowell-Mansouri as the parent action

$$\boxed{S^{(2)}_{MM}[h,\omega] = \frac{1}{2\lambda^2}\int_{AdS_D} \epsilon_{abcd\,k[D-4]}\,(\bar\nabla\omega^{ab} + 2\lambda^2\,\bar e^a h^b)(\bar\nabla\omega^{cd} + 2\lambda^2\,\bar e^c h^d)\,\bar e^{k_1}\ldots\bar e^{k_{D-4}}}$$

where

- $\bar\nabla := d + \bar\omega$ background (AdS_D) Lorentz connection
- $\bar R^{ab} := d\bar\omega^{ab} + \bar\omega^a{}_c\,\bar\omega^{cb} + \lambda^2\,\bar e^a\,\bar e^b = 0 = d\bar e^a + \bar\omega^a{}_b\,\bar e^b =: \bar T^a$

AdS_{d+1} : zero-curvature of $so(2,d)$

Gauge symmetries :

$$\begin{cases} \delta_\epsilon h^a = \bar\nabla\epsilon^a - \bar e_b\,\epsilon^{ab} \\ \delta_\epsilon\omega^{ab} = \bar\nabla\epsilon^{ab} + 2\lambda^2\,\bar e^{[a}\,\epsilon^{b]} \end{cases}$$

• Eliminate h^a via $\frac{\delta S^{(2)}_{MM}}{\delta h^a} \approx 0$ as h is auxilliary when $\lambda \neq 0$

$$\hookrightarrow \quad h_\mu{}^a \approx -\frac{1}{(D-2)\lambda^2}\left(R_\mu{}^a - \frac{1}{2(D-1)}\bar{e}_\mu{}^a R\right) =: -\frac{1}{\lambda^2} P_\mu{}^a$$

where $\underbrace{R_{\mu\nu}{}^{ab}}_{\text{Riemann-like}} := 2\bar{\nabla}_{[\mu}\omega_{\nu]}{}^{ab}$, $\underbrace{R_\mu{}^a}_{\text{Ricci-like}} := R_{\mu\nu}{}^{ab}\,\bar{e}^\nu{}_b$, $\underbrace{R}_{\text{curvature scalar-like}} := R_\mu{}^a\,\bar{e}^\mu{}_a$

and $P_\mu{}^a$ the Schouten-like tensor.

<u>Note</u> : $R_{\mu\nu|\alpha\beta} := R_{\mu\nu}{}^{ab}\,\bar{e}_{\alpha a}\,\bar{e}_{\beta b} \neq R_{\alpha\beta|\mu\nu}$

Finds

$$\boxed{S_\lambda[\omega;\bar{e}] = -\frac{(D-4)!}{2\lambda^2}\int_{AdS_D} \bar{e}\, d^Dx\; C_{cd}{}^{ab}(\omega)\, C_{ab}{}^{cd}(\omega)}$$

$C_{\mu\nu}{}^{ab} := R_{\mu\nu}{}^{ab} - 4\,\bar{e}^{[a}_{[\mu}\, P^{b]}_{\nu]}$ i.e. $C^{ab} = R^{ab} - 2\,\bar{e}^{[a}\wedge P^{b]}$ Weyl-like tensor.

• Equations of motion :

Cotton-like tensor

$$\frac{\delta S_\lambda}{\delta \omega_\mu{}^{ab}} \approx 0 \quad \Leftrightarrow \quad \tilde{C}_{ab|}{}^{\mu} := \frac{1}{D-3} \bar{\nabla}_\nu C_{ab}{}^{\mu\nu} \approx 0 \qquad (*)$$

One has $\tilde{C}_{ab|c} = 2\bar{\nabla}_{[b} P_{a]c} - 2\lambda^2 \omega_{[a|b]c}$

1) $(*)$ $\Leftrightarrow$ to spin-2 massless field in AdS_D : use unfolding techniques.

2) Zero-torsion condition for the geometric vielbein

• $\tilde{e}_\mu{}^a(\omega;\bar{e}) := \bar{e}_\mu{}^a - \frac{1}{\lambda^2} P_\mu{}^a(\omega) + \mathcal{O}(\omega^2)$

and full connexion

• $\nabla = d + w \,, \quad w = \varpi + \omega$

Indeed : $0 \approx T_{\mu\nu}{}^a := 2\nabla_{[\mu}\tilde{e}_{\nu]}{}^a = \underbrace{\bar{T}_{\mu\nu}{}^a}_{\equiv\, 0 \text{ in AdS}} + \frac{1}{\lambda^2}\tilde{C}_{\mu\nu|}{}^a + \mathcal{O}(\omega^2)$

• The child action inherits the gauge symmetries $\delta_{\epsilon}\, \omega_{\mu}{}^{ab} = 2\lambda^{2}\, \bar{e}^{[a}\, \epsilon^{b]}$

therefore, dualizing $U_{\mu[D-2]|\nu} := \frac{1}{2}\, \epsilon_{\mu[D-3]\alpha\beta}\, \omega_{\nu}{}^{\alpha\beta}$,

only the $Y_{GL_D}[D-2,1]$ part of $U_{\mu[D-2]|\nu}$ survives in $S_{\lambda}[U]$, so in AdS_D ,

dual graviton seems to be $U_{\mu[D-2]|\nu} \sim$  $\sim Y_{GL_D}[D-2,1]$

↳ one box too much in the first column compared to its flat cousin T.

Resolution

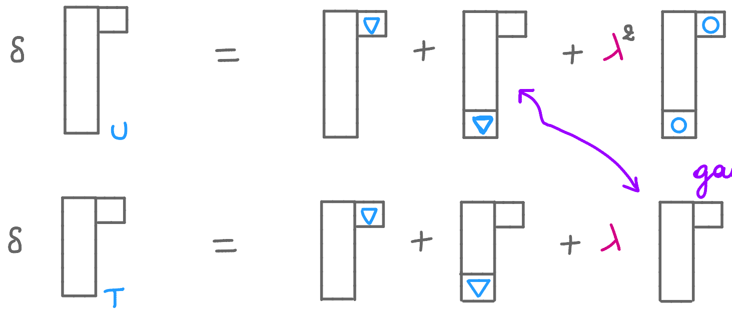
- We know [Brink - Metsaev - Vasiliev, N.B. - Iazeolla - Sundell, Alkalaev - Grigoriev]

that the theory for the gauge field $U_{[D-2,1]}$ with (maximal) symmetry

in AdS_D

$[D-2\,,\;1]$

can be extended, adding Stückelberg field $T_{[D-3,1]}$, to



gauge param. $\sigma_{[D-3,1]}$

$[D-2\,,\;1]$

in AdS_D

- In the flat $\lambda \to 0$ limit, the gauge field $U_{[D-2,1]}$ gives the sum of two theories:

$$S_{\lambda=0} = S^{\text{Curt.}}_{\lambda=0}[U_{[D-2,1]}] + S^{\text{Curt.}}_{\lambda=0}[T_{[D-3,1]}]$$

- Due to size of $U_{[D-2,1]}$, its field equations make it topological

(Labastida-Skvortsov fields require $c_1 + c_2 \leqslant D-2$ to be dynamical)

and only the field $T_{[D-3,1]}$ propagates $\longrightarrow$ recognized dual graviton.

- The MacDowell-Mansouri is 1 parent action for fully nonlinear gravity.
- The other is proposed :

$$S[\omega, e] = -\frac{1}{2\lambda^2}\int_{M_D} \epsilon_{abcd\,k[D-4]}\, C^{ab} \wedge C^{cd} \wedge e^{k} \wedge \dots \wedge e^{k} \qquad (*)$$

where now, $C^{ab}(e,\omega)$ is the full nonlinear Weyl 2-form

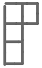
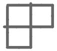
$$C^{ab} = R^{ab} - 2\, e^{[a} P^{b]} \quad , \text{ for } P_\mu{}^a \text{ complete Schouten}$$

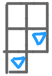
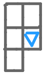
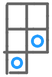
Rem : This is not conformal gravity, since in $(*)$, $\omega_\mu{}^{ab}$ independent.

In particular, even for $D=4$, $(*)$ is not invariant under

$$\delta_\sigma \omega_\mu{}^{ab} = 2\, e_\mu{}^{[a} e^{b]\nu} \partial_\nu \sigma \quad , \quad \delta_\sigma e_\mu{}^a = \sigma(x)\, e_\mu{}^a$$

THANK YOU FOR YOUR ATTENTION !

$\underline{D=5}$ $\quad U \sim$  , $\quad T \sim$ 

$K_{\lambda}^{[3,2]} =$  $+ \lambda$  $+ \lambda^2$ 

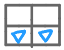
<u>eom</u> : $\mathrm{Tr}\, K_{\lambda}^{[3,2]} = 0$

$$K_{\lambda}^{[3,2]} \xrightarrow[\lambda \to 0]{} K_{0}^{[3,2]}(T) := d^{(1)} d^{(2)} T^{[2,1]}$$

twisted-duality

Pseudo-duality in AdS_5

$K_{\lambda}^{[3,2]} = *_1 K_{0}^{[2,2]}(h)$ where $K_{0}^{[2,2]}(h)$ is the gauge-invariant Linearised Riemann tensor around AdS_D .

$K =$  $+ \lambda^2$