Duality in Double and Exceptional Field Theory

Olaf Hohm

Simons Center, Stony Brook University

- E. A. Bergshoeff, O.H., V. A. Penas, F. Riccioni, 1603.07380
- O.H., H. Samtleben: 1308.1673, 1406.3348, 1410.8145

Workshop *Dualities in Supergravities, Strings and Branes*, Texas A & M University,

April 2016

Plan of the talk:

- Motivation: dual fields, branes and exotic branes
- Review: dualization of p-forms and (linearized) gravity
- Exotic dualization: Kalb-Ramond 2-form to $(D-2,2)$ Young tableaux
- Dualization of (linearized) DFT
- Dual fields in EFT
- Conclusions and Outlook

Motivation: dual fields, branes and exotic branes

- p-form gauge potential A_p dual to $(D-p-2)$-form $\tilde{A}_{D-p-2}$

$$F_{p+1} = \mathrm{d}A_p \quad \rightarrow \quad \tilde{F}_{D-p-1} \equiv \star F_{p+1} \equiv \mathrm{d}\tilde{A}_{D-p-2}$$

- Dualization essential for brane word-volume theories

  E.g. Kalb-Ramond 2-form $B_2 \rightarrow B_6$

$$\text{NS5 brane:} \qquad S_{\text{NS5}} = \int_{\Sigma_6} B_6$$

- Under T-duality $O(d,d)$ $B_2 \leftrightarrow$ metric

  $O(d,d)$ in DFT manifest (B_2 and g unified) $\rightarrow$ dual graviton

- dual graviton needed for world-volume dynamics of Kaluza-Klein monopole

  [Eyras, Janssen, Lozano (1998)]

  Generally, various mixed-Young tableaux fields expected by dualities

  $\Rightarrow$ exotic branes [Bergshoeff, Riccioni et al. (2010–), de Boer & Shigemori (2012)]

Reminder: off-shell dualization of p-forms

Action for p-form gauge field, $F_{p+1} = \mathrm{d}A_p$,

$$S[A] \;=\; -\frac{1}{2(p+1)!}\int d^Dx\, F_{a_1\ldots a_{p+1}}\, F^{a_1\ldots a_{p+1}}$$

equivalent to first-order (master) action

$$S[\tilde{A},F] \;=\; \frac{1}{(p+1)!}\int d^Dx\Big(-\frac{1}{2}F_{a_1\ldots a_{p+1}}F^{a_1\ldots a_{p+1}} - \tilde{A}^{a_1\ldots a_{p+2}}\partial_{a_1}F_{a_2\ldots a_{p+2}}\Big)$$

with gauge symmetry

$$\delta\tilde{A}^{a_1\ldots a_{p+2}} = \partial_a\Lambda^{aa_1\ldots a_{p+2}}\,, \qquad \delta F \;=\; 0$$

- EOM for $\tilde{A}$ $\;\Rightarrow\;$ Bianchi $\;\Rightarrow\;$ $F_{p+1} = \mathrm{d}A_p$
- EOM for F $\;\Rightarrow\;$ $F^{a_1\ldots a_{p+1}} \;=\; \partial_b\tilde{A}^{ba_1\ldots a_{p+1}}$

  $\Rightarrow$ integrating out F yields dual theory for $\tilde{A}$

  (note: no Levi-Civita symbol $\rightarrow$ multivector rather than form)

Dualization of linearized Einstein gravity

Linearized Einstein-Hilbert action for vielbein fluctuation $h_{a|b}$

$$S_{\mathsf{EH}}[h] \;=\; \int d^D x \left[f_{ab}{}^{b} f^{ac}{}_{c} - \tfrac{1}{2} f_{abc} f^{acb} - \tfrac{1}{4} f_{abc} f^{abc} \right]$$

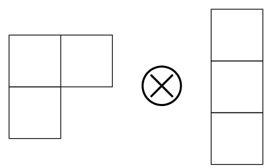
linearized coefficients of anholonomy $f_{ab}{}^{c} \;=\; 2\partial_{[a} h_{b]|}{}^{c}$

First-order (master) action [West (2001), Boulanger, Cnockaert & Henneaux (2003)]

$$S[f,D] \;=\; \int d^D x \left(f_{ab}{}^{b} f^{ac}{}_{c} - \tfrac{1}{2} f_{abc} f^{acb} - \tfrac{1}{4} f_{abc} f^{abc} + 3 D^{abc}{}_{d}\, \partial_a f_{bc}{}^{d} \right)$$

- EOM for $D^{abc}{}_{d}$ $\Rightarrow$ Bianchi $\partial_{[a} f_{bc]}{}^{d} \;=\; 0$ $\Rightarrow$ linearized gravity
- EOM for $f_{ab}{}^{c}$ determines $f_{ab}{}^{c} \;=\; 6\, \partial_d D^{d}{}_{[b}{}^{c}{}_{a]} + \cdots$
  yields dual theory for

$$\tilde{D}_{a_1 \ldots a_{D-3}|b} \;\equiv\; \tfrac{1}{6}\, \epsilon_{a_1 \ldots a_{D-3} cde}\, D^{cde}{}_{b} \quad \in$$



mixed Young tableau $(D-3,1)$ field: dual graviton

$\tilde{D}_{[a_1 \ldots a_{D-3}|b]}$ pure Lorentz gauge, but needed in master action

Exotic Dualization: 2-form to $(D-2,2)$ Young tableau field

First, rewrite Kalb-Ramond action [Boulanger et al. (2012–)]

$$S[b] \;=\; -\tfrac{1}{12}\int d^Dx\, H^{abc}H_{abc} \;=\; -\tfrac{1}{4}\int d^Dx\,\big(\partial^a b^{bc}\,\partial_a b_{bc} - 2\,\partial_a b^{ab}\,\partial^c b_{cb}\big)$$

First-order (master) action → promote $Q_{a|bc} \equiv \partial_a b_{bc}$ to be independent

$$S[Q,D] \;=\; \int d^Dx\big(-\tfrac{1}{4}\,Q^{a|bc}Q_{a|bc} + \tfrac{1}{2}\,Q_{a|}{}^{ab}Q^{c|}{}_{cb} - \tfrac{1}{2}\,D^{ab|cd}\,\partial_a Q_{b|cd}\big)$$

where $Q_{a|bc} = -Q_{a|cb}\,,\quad D_{ab|cd} = -D_{ba|cd} = -D_{ab|dc}$

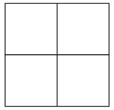
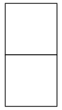
Gauge symmetries

$$\delta Q_{a|bc} \;=\; \partial_a K_{bc}\,, \qquad K_{ab} \;\equiv\; 2\partial_{[a}\tilde{\xi}_{b]}\,,$$
$$\delta D_{ab|cd} \;=\; \partial^e \Sigma_{eab|cd} + 4\,\eta_{[a[\underline{c}}K_{b]\underline{d}]}$$

where $\Sigma_{abc|de} \equiv \Sigma_{[abc]|de} \equiv \Sigma_{abc|[de]}$, $\tilde{\xi}_a$ Kalb-Ramond parameter

Decomposing dual field ($D = 4$ for simplicity)

$$D_{ab|cd} \;=\; \tfrac{1}{2}\epsilon_{ab}{}^{ef}B_{ef,cd} + 4\eta_{[a[\underline{c}}C_{b]|\underline{d}]} - 2\eta_{c[a}\eta_{b]d}C$$

where $B \;\in$  and $C_{a|b} \;\in$  $\oplus$ 

Gauge transformations $\Big[\, \Sigma_{abc|ef} \equiv \epsilon_{abc}{}^{d}\tilde{\Sigma}_{ef|d}\,, \quad \tilde{\Sigma}_{ab|c} = \lambda_{ab,c} + \epsilon_{abcd}\,\xi^{d} \Big]$

$$\delta B_{ab,cd} \;=\; \partial_{[a}\lambda_{|cd|,b]} + \partial_{[c}\lambda_{|ab|,d]}$$

$$\delta C_{a|b} \;=\; 2\partial_{[a}\tilde{\xi}_{b]} - \partial_{b}\xi_{a} + \tfrac{1}{4}\epsilon_{a}{}^{cde}\partial_{c}\lambda_{de,b}$$

Duality relations gauge invariant

$$Q_{b|cd} \;=\; -\tfrac{1}{3!}\epsilon_{b}{}^{efg}\,\Gamma_{efg,cd} \;+\; 2\partial_{[c}C_{|b|d]} \qquad \Gamma_{abc,de} \;\equiv\; 3\,\partial_{[a}B_{bc],de}$$

Integrability condition: act with $\epsilon^{abij}\partial_{a}$, using $\partial_{[a}Q_{b]|cd} = 0$

$$0 \;=\; -\partial_{a}\Gamma^{aij}{}_{,cd} + 2\epsilon^{abij}\partial_{a}\partial_{[c}C_{|b|d]} \quad\Leftrightarrow\quad \mathcal{R}^{abc}{}_{,abc} \;\equiv\; 3\,\partial_{[a}\Gamma^{abc}{}_{,bd]} \;=\; 0$$

$\Rightarrow B$ consistently propagates single d.o.f., no action for $(2,2)$ alone

Exotic Dual Action for $(2,2)$ Field

Decomposing into symmetric and antisymmetric part, $C_{a|b} = s_{ab} + a_{ab}$,

$$\mathcal{L} \;=\; -\tfrac{1}{24}\,\Gamma^{abc,de}\,\Gamma_{abc,de} + \tfrac{1}{12}\,\epsilon^{abcd}\,\Gamma_{bcd,}{}^{ef}\,h_{aef} - \tfrac{1}{6}\,\epsilon^{abcd}\,\Gamma_{bcd,}{}^{ef}\,\gamma_{ae,f} + s^{ab}G_{ab}(s) + \tfrac{1}{6}\,h^{abc}h_{abc}$$

where G_{ab} is 'Einstein tensor' for s_{ab}, and

$$\gamma_{ab,c} \;\equiv\; \tfrac{1}{2}(\partial_a s_{bc} + \partial_b s_{ac} - \partial_c s_{ab})\;, \qquad h_{abc} \;\equiv\; 3\partial_{[a}a_{bc]}$$

$\Rightarrow$ *wrong-sign* Einstein-Hilbert and Kalb-Ramond terms!

(linearized diff and KR gauge invariant with parameters ξ_a and $\tilde{\xi}_a$)

However, off-diagonal action with enlarged gauge invariance
$\Rightarrow$ no contradiction with equivalence to single 2-form

(Linearized) Double Field Theory

Frame formulation of DFT

$$\mathcal{H}_{MN} \;=\; \mathcal{E}_M{}^A\,\mathcal{E}_N{}^B\,S_{AB}\,, \qquad S_{AB} \;\equiv\; \begin{pmatrix} \eta^{ab} & 0 \\ 0 & \eta_{ab} \end{pmatrix}$$

where A local Lorentz index for $O(D-1,1)\times O(D-1,1)$

Linearized action for $\mathcal{E}_A{}^M = \bar{\mathcal{E}}_A{}^M + h_A{}^B\,\bar{\mathcal{E}}_B{}^M$

$$S_{\rm DFT} \;=\; \int d^{2D}X\, e^{-2\bar{d}}\left(S^{AB}\mathcal{F}_A\mathcal{F}_B + \frac{1}{6}\,\bar{\mathcal{F}}^{ABC}\mathcal{F}_{ABC}\right)$$

with ‘coefficients of anholonomy’

$$\mathcal{F}_{ABC} \;=\; 3\,\partial_{[A}h_{BC]} \qquad \mathcal{F}_A \;=\; \partial^B h_{BA} + 2\,\partial_A d$$

Bianchi identities

$$\begin{aligned} \partial_{[A}\mathcal{F}_{BCD]} &= 0 \\ \partial^C\mathcal{F}_{CAB} + 2\,\partial_{[A}\mathcal{F}_{B]} &= 0 \\ \partial^A\mathcal{F}_A &= 0 \end{aligned}$$

Master Action & Duality Relations for DFT

Master action introducing Lagrange multiplier for Bianchi identities

$$S \;=\; \int dX\, e^{-2\bar{d}} \Big[S^{AB}\mathcal{F}_A\mathcal{F}_B + \frac{1}{6}\,\bar{\mathcal{F}}^{ABC}\mathcal{F}_{ABC}$$
$$+\, D^{ABCD}\,\partial_A\mathcal{F}_{BCD} + D^{AB}\big(\partial^C\mathcal{F}_{CAB} + 2\,\partial_A\mathcal{F}_B\big) + D\,\partial^A\mathcal{F}_A\Big]$$

Varying w.r.t. $\mathcal{F}$ $\rightarrow$ duality relations

$$\bar{\mathcal{F}}^{ABC} \;=\; 3\,\big(\partial_D D^{DABC} + \partial^{[A}D^{BC]}\big)$$
$$2S^{AB}\mathcal{F}_B \;=\; 2\,\partial_B D^{BA} + \partial^A D$$

Breaking $O(D,D) \supset GL(D)$ and $O(D-1,1)^2$ to diagonal subgroup $A = (^a,{}_a)$

$$D_{ABCD} \;\rightarrow\; D^{abcd} \quad D^{abc}{}_d \quad D^{ab}{}_{cd} \quad D^a{}_{bcd} \quad D_{abcd}$$

encodes conventional duals to metric, 2-form, dilaton, exotic duals, etc., but *extra fields* D_{AB} and D needed for master/dual action

Dual Diffeomorphisms and Dual Action for DFT

Choose basis where $A = (a, \bar{a})$, DFT fields: $h_{a\bar{b}} \;\; h_{ab} \;\; h_{\bar{a}\bar{b}} \;\; d$

dual DFT fields: $D_{abcd} \;\; D_{abc\bar{d}} \;\; \ldots \;\; D_{\bar{a}\bar{b}\bar{c}\bar{d}} \qquad D_{a\bar{b}} \;\; D_{ab} \;\; D_{\bar{a}\bar{b}} \;\; D$

Dual diffeomorphisms

$$\begin{aligned} \delta_{\Sigma} D_{a\bar{b}} &= \partial_a \Sigma_{\bar{b}} - \partial_{\bar{b}} \Sigma_a + \partial^c \Sigma_{ca,\bar{b}} + \partial^{\bar{c}} \Sigma_{\bar{c}\bar{b},a} \\ \delta_{\Sigma} D &= \partial_a \Sigma^a + \partial_{\bar{a}} \Sigma^{\bar{a}} \\ \delta_{\Sigma} D_{ab\bar{c}\bar{d}} &= 2\,\partial_{[a}\, \Sigma_{|\bar{c}\bar{d}|,b]} - 2\,\partial_{[\bar{c}}\, \Sigma_{|ab|,\bar{d}]} \qquad \cdots \end{aligned}$$

Dual DFT Action

$$\begin{aligned} \mathcal{L} \;=\; & -\tfrac{1}{2}\,\partial_{\bar{d}} D^{\bar{b}\bar{c}\bar{d},a}\,\partial^{\bar{e}} D_{\bar{b}\bar{c}\bar{e},a} - \partial_{\bar{d}} D^{\bar{b}\bar{c}\bar{d},a}\,\partial^{e} D_{ea,\bar{b}\bar{c}} - \tfrac{1}{2}\,\partial_{d} D^{da,\bar{b}\bar{c}}\,\partial^{e} D_{ea,\bar{b}\bar{c}} \\ & + \text{three more lines} \\ & - D^{ab,\bar{c}\bar{d}}\,\mathcal{R}_{ab,\bar{c}\bar{d}}(D) - \mathcal{L}^{(2)}_{\text{DFT}}(D_{a\bar{b}}, D) \end{aligned}$$

where $\mathcal{R}_{ab,\bar{c}\bar{d}} \equiv 4\,\partial_{[a}\partial_{[\bar{c}}\, D_{b]\bar{d}]}$ linear DFT Riemann tensor

Dual Fields in $E_{8(8)}$ Exceptional Field Theory

$E_{8(8)}$ vielbein $\mathcal{V}_M{}^{\underline{M}}$, coordinates Y^M, $M = 1, \ldots, 248$, subject to

$$\mathbb{P}_{MN}{}^{KL} \equiv \left(\mathbb{P}_{1+248+3875}\right)_{MN}{}^{KL} \qquad \mathbb{P}_{MN}{}^{KL}\partial_K \otimes \partial_L \;=\; 0$$

Naive generalized Lie derivative

$$\mathbb{L}_\Lambda V^M \;\equiv\; \Lambda^K \partial_K V^M - f^M{}_{NP}\, f^{PK}{}_L\, \partial_K \Lambda^L\, V^N$$

Two (related) puzzles:

1) do not form an algebra [Berman, Cederwall, Kleinschmidt & Thompson (2013)]

2) $\mathcal{V}_M{}^{\underline{M}} \;\supset\; \varphi_m$ $(m = 1, \ldots, 8)$ dual to KK vector $A_\mu{}^m$
dual graviton ($C_{m_1 \ldots m_8, n} = \epsilon_{m_1 \ldots m_8} \varphi_n$) *not* in $D = 11$

However, gauge invariant ‘Ricci scalar’ exists ($\mathcal{M} = \mathcal{V}\mathcal{V}^T$):

$$\mathcal{R}(\mathcal{M}) \;=\; -\frac{1}{240}\mathcal{M}^{MN}\partial_M \mathcal{M}^{KL}\, \partial_N \mathcal{M}_{KL} + \frac{1}{2}\mathcal{M}^{MN}\partial_M \mathcal{M}^{KL}\partial_L \mathcal{M}_{NK}$$
$$+ \frac{1}{7200}\, f^{NQ}{}_P f^{MS}{}_R\, \mathcal{M}^{PK}\partial_M \mathcal{M}_{QK} \mathcal{M}^{RL}\partial_N \mathcal{M}_{SL} + \cdots$$

Resolution of puzzle: additional gauge symmetry

$$\delta_{(\Lambda,\Sigma)} V^M \equiv \Lambda^K \partial_K V^M - f^M{}_{NP} f^{PK}{}_L \partial_K \Lambda^L V^N \underline{- \Sigma_L f^{LM}{}_N V^N}$$

$\Rightarrow$ closure & invariance of action provided *covariantly constrained*

$$\mathbb{P}_{MN}{}^{KL} \partial_K \otimes \Sigma_L = \mathbb{P}_{MN}{}^{KL} \Sigma_K \otimes \Sigma_L = 0$$

$\Rightarrow$ 8 components upon solving constraint $\Rightarrow$ $\delta\varphi_m = \lambda_m$, pure gauge

$\Rightarrow$ corresponding gauge vectors $B_{\mu M}$ needed for Chern-Simons terms

General tensor hierarchy structure for any $\mathrm{E}_{n(n)}$

$D=3$, $\mathrm{E}_{8(8)}$: $\quad \mathcal{D}_\mu \mathcal{V} = (\partial_\mu - \delta_{(A_\mu, B_\mu)}) \mathcal{V}\,, \quad \delta B_{\mu M} = \partial_\mu \Sigma_M + \cdots$

$D=4$, $\mathrm{E}_{7(7)}$: $\quad \mathcal{F}_{\mu\nu}{}^M = 2\partial_{[\mu} A_{\nu]}{}^M + \cdots - \frac{1}{2}\Omega^{MN} B_{\mu\nu N}$

$D=5$, $\mathrm{E}_{6(6)}$: $\quad \mathcal{H}_{\mu\nu\rho M} = 3\partial_{[\mu} B_{\nu\rho]M} + \cdots + C_{\mu\nu\rho M}$

Compensator gauge fields needed for supersymmetry

[Godazgar[2], O.H., Nicolai, Samtleben (2014)]

representations not part of E_{10} or E_{11} level decomposition

Generalized Scherk-Schwarz & Consistent KK Truncations

Ansatz in terms of $U(Y) \in E_7$

$$\mathcal{M}_{MN}(x,Y) = U_M{}^{\underline{K}}(Y)\, U_N{}^{\underline{L}}(Y)\, M_{\underline{K}\,\underline{L}}(x)$$

$$\mathcal{A}_\mu{}^M(x,Y) = \rho^{-1}(Y)(U^{-1})_N{}^M(Y)\, A_\mu{}^N(x)$$

Y drops out $\Leftrightarrow$ *consistency (twist) conditions* for $\mathcal{E}_{\underline{M}}{}^N \equiv \rho^{-1}(U^{-1})_{\underline{M}}{}^N$

[Aldazabal, Grana, Marques, Rosabal (2013)]

$$\mathbb{L}_{\mathcal{E}_{\underline{M}}}\mathcal{E}_{\underline{N}} = -X_{\underline{MN}}{}^{\underline{K}}\,\mathcal{E}_{\underline{K}}\,, \qquad X_{\underline{MN}}{}^{\underline{K}} = \Theta_{\underline{M}}{}^\alpha (t_\alpha)_{\underline{N}}{}^{\underline{K}} + \cdots$$

Full consistency: *non-trivial* ansatz for compensating gauge field

$$\mathcal{B}_{\mu\nu\, M}(x,Y) = -2\,\rho^{-2}\,(U^{-1})_S{}^P \partial_M U_P{}^R (t^\alpha)_R{}^S\, B_{\mu\nu\,\alpha}(x)$$

Twist matrices constructed for $AdS_4 \times S^7$, $AdS_5 \times S^5$, hyperboloids, etc.,

$\Rightarrow$ Consistency of KK truncation [de Wit & Nicolai (1986)]

Outlook & Summary

- Dual graviton and other mixed Young tableau fields in DFT
  $\Rightarrow$ new features, new fields
- Construction of *non-linear* completion in completely new light, no-go theorems not applicable
- Fully non-linear DFT?
- Parts of dual graviton appear consistently and non-linearly in EFT, needed for supersymmetry and consistent KK ansatz
- Application for exotic branes?